ELASTIC AND ELASTO-PLASTIC BUCKLING ANALYSIS OF PERFORATED STEEL PLATES

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ABSTRACT

Many steel structures such as ships and offshore structures are composed by welded stiffened or unstiffened plate elements. Cutouts are often provided in these plate elements for inspection, maintenance, and service purposes, and the size of these holes could be significant. In many situations, these plates are subjected to axial compressive forces which make them prone to instability or buckling. If the plate is slender, the buckling is elastic. However, if the plate is sturdy, it buckles in the plastic range causing the so-called inelastic (or elasto-plastic) buckling. Furthermore, the presence of these holes redistributes the membrane stresses in the plate and may cause significant reduction in its strength in addition to changing its buckling characteristics. So, the objective of this paper is to investigate the changes that the presence of circular holes produces in the elastic and inelastic buckling of steel rectangular plates. The finite element method (FEM) has been used to evaluate the elastic and elasto-plastic buckling load of uniaxially loaded rectangular plates with circular cutouts. By varying the hole diameter, the plate aspect ratio and the plate thickness during the analyses, the changes in the plate buckling behavior can be determined. The results show that while the circular hole can in some cases even increase the elastic buckling load, the elasto-plastic buckling load is reduced by the presence of the cutout.

KEY WORDS: Elastic buckling. Elasto-plastic buckling. Finite element method. Numerical simulation.

1. INTRODUCTION

Many thin-walled structures contain holes. In marine and offshore structures, the perforated panels are used to make a way of access or to reduce the total weight of the structure. When these plates are subject to compression loads, the structure could buckle if the load exceeds the critical load. Thus, to know how this phenomenon occurs and to analyze the buckling behavior of these perforated panels has great importance for an efficient structural design. An example of a ship hull with circular holes is shown in FIGURE 1.

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The elastic buckling is an instability phenomenon that can occur if a slender and thin-walled plate (plane or curved) is subjected to axial compression. At a certain given critical load the plate will suddenly bend in the out-of-plane transverse direction. However, if the plate is sturdy, it buckles in the plastic range causing the so-called inelastic (or elasto-plastic) buckling.



Figure 1. Circular holes in the ship hull.

The buckling behavior of perforated plates has been the object of a large number of researches in the last decade. The main objective of the published articles can be divided into two categories, i.e., elastic buckling and elasto-plastic buckling. Among the elastic buckling studies category, El-Sawy and Nazmy [2] investigated the effect of aspect ratio on the elastic buckling critical loads of uniaxially loaded rectangular plates with eccentric circular and rectangular (with curved corners) holes. El-Sawy and Martini [4] used the finite element method to determine the elastic buckling stresses of biaxially loaded perforated rectangular plates with longitudinal axis located circular holes. Alternatively, Moen and Schafer [5] developed, validated and summarized analytical expressions for estimating the influence of single or multiple holes on the elastic buckling critical stress of plates in bending or compression.

In the group of studies dedicated to the problem of elasto-plastic buckling, El-Sawy et al. [3] investigated the elasto-plastic buckling of uniaxially loaded square and rectangular plates with circular cutouts by the use of the finite element method, including some recommendations about hole size and location for the perforated plates of different aspect ratios and slenderness ratios. Afterwards, Paik [6,7,8] studied the ultimate strength characteristics of perforated plates under edge shear loading, axial compressive loading

and the combined biaxial compression and edge shear loads, and proposed closed-form empirical formulae for predicting the ultimate strength of perforated plates based on the regression analysis of the nonlinear finite element analyses results.

So, the objective of this paper is to investigate the changes that the presence of circular holes produces in the elastic and inelastic buckling of steel rectangular plates. The finite element method (FEM) has been used to evaluate the elastic and elasto-plastic buckling load of uniaxially loaded rectangular plates with circular cutouts. By varying the hole diameter, the plate aspect ratio and the plate thickness during the analyses, the changes in the plate buckling behavior can be determined.

2. METHOD OF ANALYSIS

The objective of this work is to study the elastic and the inelastic buckling behavior of perforated rectangular thin plates under uniaxial compression loading as it can be seen in FIGURE 2.

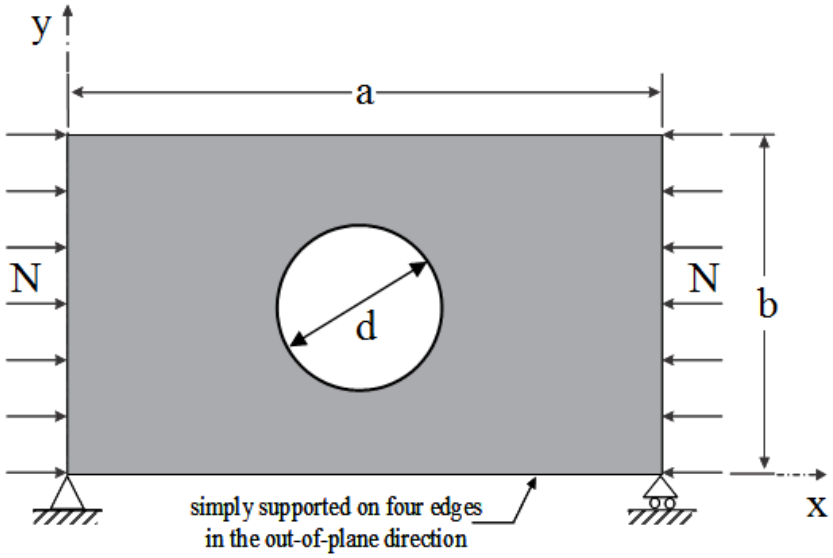

Figure 2. Plate with centered circular hole subject to uniaxial compression.

The approach adopted for the elastic buckling analysis was the eigenvalue buckling (linear). This numerical procedure is used for calculating the theoretical buckling load of a linear elastic structure. Since it assumes the structure exhibits linearly elastic behavior, the predicted buckling loads are overestimated. Therefore, if the component is expected to exhibit structural instability, the search for the load that causes structural bifurcation is referred to as a buckling load analysis. Because the buckling load is not known a priori,

the finite element equilibrium equations for this type of analysis involve the solution of homogeneous algebraic equations whose lowest eigenvalue corresponds to the buckling load, and the associated eigenvector represents the primary buckling mode [9]. In the finite element program ANSYS®, the eigenvalue problem is solved by using the Lanczos numerical method [1].

On the other hand, the determination of the inelastic buckling stress for perforated plates requires a more sophisticated analysis since the initial stress stiffness matrix, kg, is not proportional to the stress level in the plate anymore due to the geometric and material nonlinearities. A general-purpose finite element program, ANSYS® has been utilized in this investigation [1]. The plate material was assumed to be linear elastic–perfectly plastic, which is the most critical case for the steel material. An initial imperfect geometry that follows the buckling mode of an elastic eigenvalue pre-analysis is assumed. The maximum value of the imperfection is chosen to be b/2000, where b is the perforated plate width. The uniaxial load is gradually applied until the convergence cannot be attained anymore by Newton-Raphson method. The finite element analysis used is capable of modeling the material nonlinearity as well as the geometric nonlinearity due to large deformations and small strains.

3. VERIFICATION OF THE METHOD OF ANALYSIS

For the first verification of the computational modeling, the critical load of a non perforated plate was numerically evaluated, and the result was compared with the analytical solution given by Timoshenko and Gere [10]. The main characteristics of the analyzed plate are shown in TABLE 1.

Table 1. Characteristics of the solid plate.

Characteristic	Value
Young's modulus *(E)*	210.0 GPa
Poisson's ratio (*v*)	0.3
width of plate (*H*)	1.0 m
length of plate (*L*)	2.0 m
thickness of plate (*t*)	10.0 mm

In all numerical simulations the ANSYS® SHELL93 reduced integration eight-node thin shell element was employed. This element has six degrees-of-freedom at each node: three translations (*u, v, w*) and three rotations (θ_x, θ_y, θ_z). The plate was discretized

adopting a triangular element with side size of 50.00 mm (*b/20*), generating a mesh with 1814 finite elements (FIGURE 3(a)). The numerical result for the critical buckling load N_{cr} is 755.30 kN/m, showing a difference of -0.51% comparing with the analytical solution that is equal to 759.20 kN/m. FIGURE 3(b) presents the buckled shape of the plate without hole.

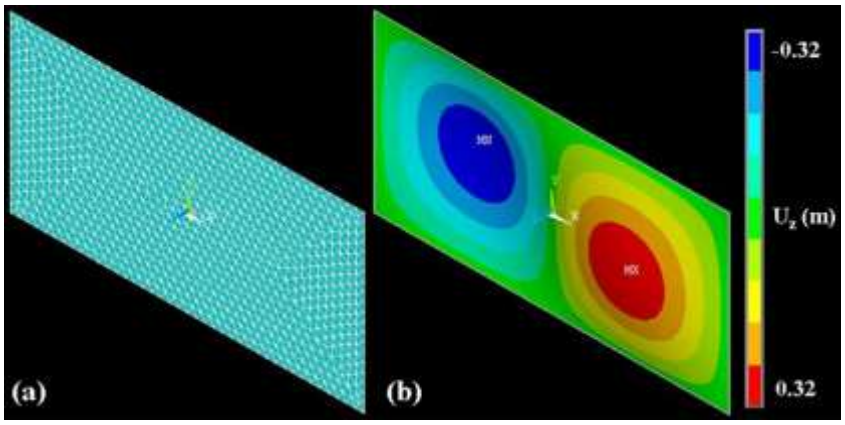

Figure 3. Plate without hole: (a) Finite element mesh; (b) Buckled shape.

However, for plates with perforations there is no analytical solution available and the approach adopted for buckling analysis was the finite element eigenvalue buckling analysis. Here, the computational model previously presented was employed to analyze the buckling behavior of thin perforated plates already studied by El-Sawy and Nazmy [2]. The same plate used in the first verification was studied, however centered circular holes were considered. In TABLE 2 the results for the critical buckling load were compared with those obtained by the numerical study developed by El-Sawy and Nazmy [2]. Again an excellent agreement was obtained, being -0.53% the maximal difference encountered.

Table 2. Comparison of critical buckling load for plate with centered circular hole.

Hole diameter (m)	N_{cr} (kN/m) Reference [1]	N_{cr} (kN/m) Authors	Difference (%)
0.10	766.19	763.56	-0.34
0.20	789.36	786.50	-0.36
0.30	825.08	820.87	-0.51
0.40	849.26	847.78	-0.17
0.50	901.54	898.79	-0.31
0.60	986.46	981.22	-0.53

In order to verify the method used in the nonlinear buckling analysis, a comparison with existing results in the literature on the inelastic buckling of square plates with concentric circular holes was performed. The results of El Sawy et al. [3], who also used the finite element program ANSYS to determine the inelastic buckling stress, were used for this purpose. The model used by El Sawy et al. [3] was composed of mainly four-noded shell elements that had six degrees of freedom per node. Three-noded shell elements were only used in irregular zones around the hole.

TABLE 3 shows a comparison between the authors' results using ANSYS and El Sawy et al. [3], for a square plate of thickness *t* and side length *b*, with a central circular hole of dimension *d*. The steel used in both analyses is A572 Grade 50 steel (with σ_y = 345 MPa), and the comparison is made for three values of plate slenderness ratio *b/t* = 30, 40, 50, and three values of normalized hole size (*d/b* = 0.10, 0.20, 0.30). The comparison shows an excellent agreement for almost all values and the maximum difference was 4.50%.

Table 3. Ratio between inelastic buckling stress and yielding stress for a square plate with centered circular hole.

	***d/b* = 0.10**		***d/b* = 0.20**		***d/b* = 0.3**	
	Authors	*Ref.[3]*	*Authors*	*Ref.[3]*	*Authors*	*Ref.[3]*
***b/t* = 30**	0.900	0.910	0.800	0.810	0.700	0.710
***b/t* = 40**	0.895	0.880	0.793	0.790	0.695	0.690
***b/t* = 50**	0.784	0.750	0.700	0.690	0.620	0.610

4. RESULTS AND DISCUTION

4.1 Linear Buckling Analysis of Perforated Plates

All the plates analyzed in this study have the following constant dimensions: *b* = 1.00 m and *t* = 0.01 m. The aspect ratio is variable: *a/b* = 1, 2, 3, 4 and 5. The hole diameter also varies: *d* = 0.10, 0.20, 0.30, 0.40, 0.50 and 0.60 m. Once the elastic buckling load, N_{cr}, has been evaluated, the elastic critical buckling stress σ_{cr} can be determined by dividing it by the plate thickness *t*.

The results obtained for the critical buckling stress of perforated plates are presented in FIGURE 4. The material yielding stress σ_y, was used to normalize critical stress σ_{cr} and the width *b* was adopted to normalize the hole diameter.

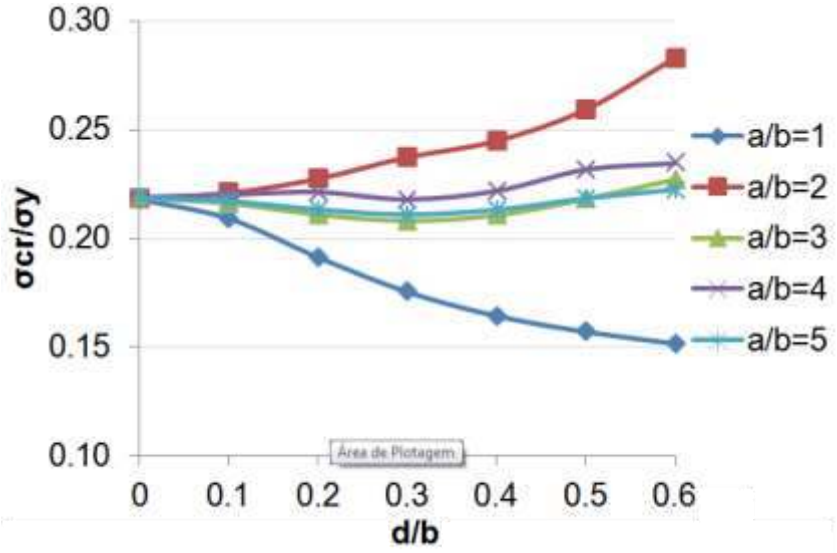

Figure 4. Ratio between the elastic critical stress σ_{cr} and the material yielding stress σ_y for perforated plates

The perforated plate with aspect ratio *a/b = 2* has the best behavior among the studied cases. There is an increase in the critical buckling stress as the hole size also increases. This trend could be explained if one considers the buckled mode shapes of the plate, which are presented in FIGURE 5. In fact, the buckling resistance increases due to a redistribution of the membrane stresses towards the laterally supported side edges of the plate (FIGURE 5 (a, b, c)). When the ratio *d/b* increases the plate buckled shape changes from two half-waves to three half-waves. This explains the increasing of the buckling load in these cases (FIGURE 5 (d, e, f)).

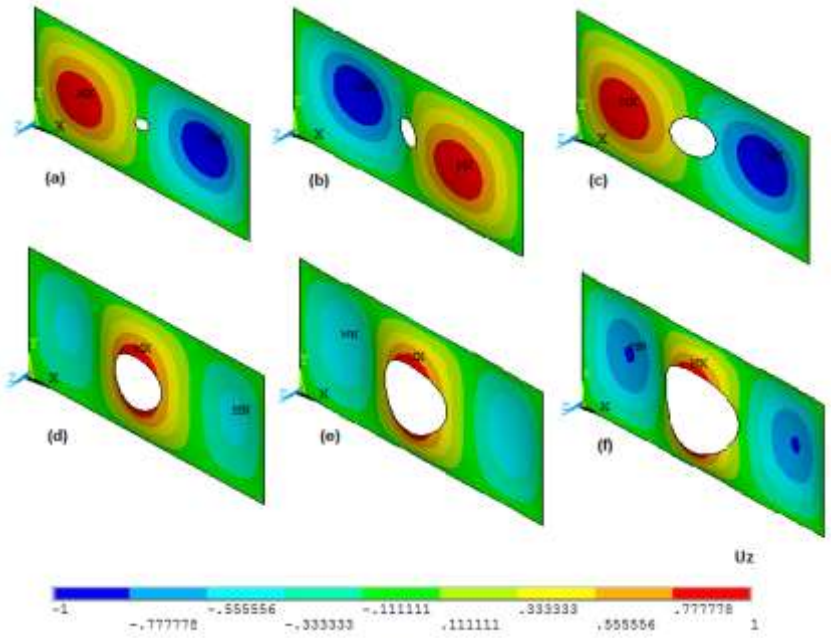

Figure 5. Buckling modes for a plate with $a/b = 2$.

4.2 Nonlinear Buckling Analysis of Perforated Plates

A series of numerical tests were conducted to investigate the buckling behavior of perforated plates both in the linear as in the nonlinear material range. As a result, both elastic and inelastic buckling stress curves were plotted against the plate slenderness ratio in order to determine the governing failure mode as a function of the plate slenderness ratio, which is a very important aspect in the design of perforated plates.

FIGURE 6 shows the behavior of the ratio between the critical buckling stress σ_{cr} and yield stress σ_y with increasing slenderness ratio b/t for square plates with varying d/b ratios (d is hole diameter and b is width of the plate). A curve dividing the linear elastic behavior from the elasto-plastic behavior is also shown in FIGURE 6.

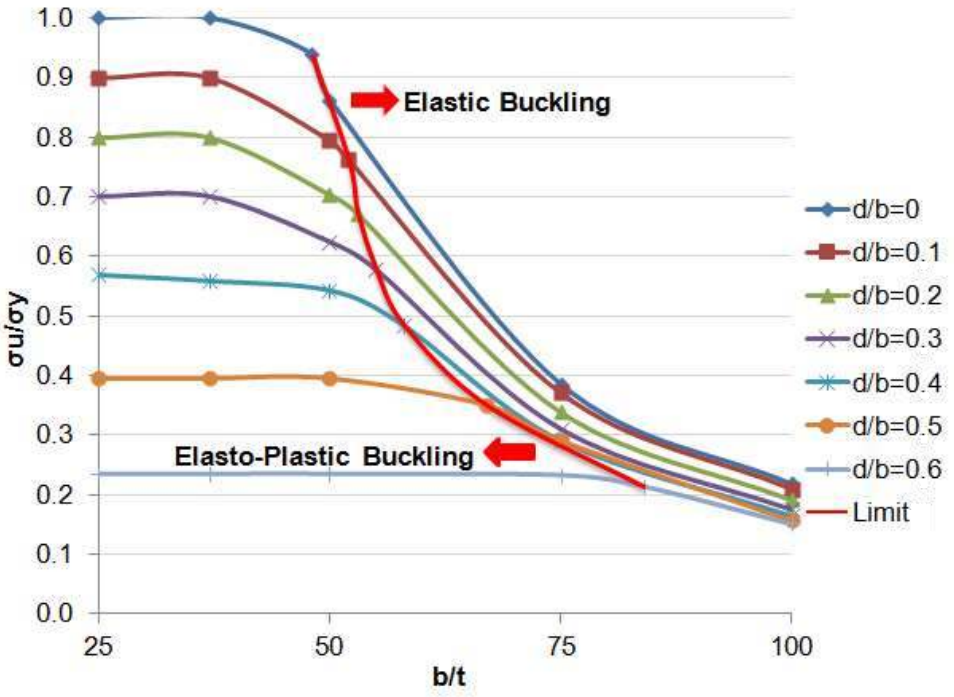

Figure 6. Buckling curves for a square plate.

The curves shown in FIGURE 6 illustrate the change in the ‘‘governing’’ critical stress (smaller of both the elastic and inelastic buckling stresses) versus the plate slenderness ratio for different values of hole sizes (d/b = 0.0–0.6). It is clear from this figure that the critical stress σ_{cr} decreases as b/t increases, and the failure mode changes from elasto-plastic to pure elastic buckling. On the other hand, the critical buckling stress decreases with the hole size, and may occur after the plate material has reached the yield point at some portions of the plate, and that is called inelastic, or elasto-plastic, buckling, especially for large size holes (d/b = 0.6).

5. CONCLUSIONS

The importance of thin perforated plates as structural members is evident in many engineering applications, especially in naval, marine and offshore structures. The failure of perforated plates subjected to uniaxial compression may be due to the plate’s out-of-plane instability or material failure. For thin perforated plates (i.e., large values of b/t), instability occurs at an average stress σ_{cr}, that is much less than the yield stress σ_y, especially if the hole size is small. This is called elastic buckling.

On the other hand, the buckling for relatively thicker plates (i.e., low *b/t* values), or plates with large holes, may occur after the plate material has reached the yield point at some portions of the plate, and that is called inelastic, or elasto-plastic, buckling.

The large utilization of perforated steel plates in the ship and offshore structures construction and the change of the failure mode of these elements with plate slenderness and hole size demonstrate that the development of additional researches on this subject are necessary.

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7. ACKNOWLEDGEMENTS

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