THERMAL OPTIMIZATION OF CIRCULAR BODIES SUBMITTED TO AN INTENSE HEAT FLUX USING CONSTRUCTAL DESIGN

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Abstract

The present work studies the optimization of a circular body with an intense heat flux by Constructal Design. The problem concerns the minimization of the global thermal resistance of a three-dimensional structure submitted to an intense uniform heat flux, which is cooled through micro-channels inserted in the circular body. For the optimization the body and the channels volumes are kept constants, while the geometrical configuration varies. Two geometric configurations were studied: radial and with one level of bifurcation - the first construct. The conservation equations of mass, momentum and energy are solved using a commercial package based on the finite volume method. For the radial configuration the system was successfully optimized as function of the number of ducts intruded into the body. For the bifurcated configuration Constructal Design led to a double optimization: one as function of the angle between the branches on the bifurcation (δ) and other as function of the ratio between the length of a single duct (L_0) and the radius of the circular domain (L).

Keywords: Thermal optimization; Circular bodies; Intense Heat Flux; Constructal Design.

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1. INTRODUCTION

Constructal Theory is based on the objective and constraints principle. According to this principle, one flow system subject to constraints will change its geometry in order to achieve the lower resistance of its internal currents. Additionally, Constructal Theory lead to the optimal geometry configuration for systems submitted to any kind of flow, such as: heat transfer, fluid flow, cars across the city, distribution of blood along the body, trees and others (Bejan, 2000). This theory is not only employed to achieve some purpose in engineering, but also to explain how the geometry of natural systems can be deterministically described.

Bejan and Lorente (2006) presented many examples of application of Constructal Theory for the generation of shape and structure in nature, for instance: rivers cross section, shape of animals, soil fractures and earth climate. Examples of engineering systems, which can be optimized according to this theory, were also presented, for instance, electronic packages, fuel cells, construction of streets and development of thermal device and equipments.

Constructal design has been employed for the optimization of the heat transfer in T- and Y-shaped fins (Bejan and Almogbel, 2000; Lorenzini and Rocha, 2006), as well as, for the minimization of the global thermal resistance of heat generating body cooled by negative or inverted fins (Biserni et al., 2004). However, few studies have been presented about the optimization of the three-dimensional bodies submitted to an intense heat flux cooled through micro-channels by means of Constructal Design. One example of this kind of application was performed by Rocha et al. (2009) which simulated the forced convection cooling of vascularised wall submitted to intense heat transfer. In this study, it was noticed that the optimal geometry represents the equilibrium between the resistance through the interstices and the one along the channel and this equilibrium led to tree shaped geometries.

The performance of the microelectronic packages has increased significantly during the last three decades. Nevertheless, this increase associated with the decrease of their dimensions has demanded ways to dissipate efficiently the heat transfer generated by these devices. Among the various methods for cooling, the micro-channels dissipater with high heat flux has been one of the most well succeeded. This technique was first introduced by Tuckerman and Pease (1981) for cooling integrated electronic packages. An overview on the application of micro-channels for dissipation of heat generation of electronic systems was presented by Sohban and Garimella (2001).

According to Bejan and Errera (1997) the geometric design of the micro-channels cooling system can mimic the structure of live organisms in order to increase its efficiency, introducing the concept of tree-shaped structures for cooling. The authors also observed that the structure of tree-shape reduces the resistance of the flux.

In the present work it is studied a three-dimensional disc-shaped body submitted to an intense constant heat flux in its lower face. This body is cooled by a fluid that flows along square section ducts inserted into the body. The fluid movement is imposed by a constant pressure difference between the inlet (on the center of the disk) and the outlet (on the periphery of the disk) of the ducts. For the optimization of the body geometry and the channels configuration is employed Constructal Design. Two configurations for the channels are studied: radial and with one level of bifurcation, which represents the first construct. The ratio between the volume of the channels and the volume of the solid body is kept fixed for all simulated cases, being the constraint of the problem. On the other hand, the number of channels (n) for the radial configuration varies. For the bifurcated geometry, the angle between the bifurcated branches (δ) as well as the ratio between the length of the single branch duct (L_0) and the radius (L) vary, being the degrees of freedom. The global performance indicator is the global thermal resistance between the solid body and the cooling channels. Additionally, for all simulated cases Bejan and Prandtl numbers for the fluid were kept fixed, Be = 10^8 and Pr = 0.71.

2. MATHEMATICAL MODEL

The analyzed problem consists of a circular body with several ducts inserted into the body, as can be seen in FIGURE 1A. The ducts can be placed in a radial or bifurcated form, as shown in FIGURE 1B and 1C, respectively. Once the domain volume is composed of several ducts disposed symmetrically, it is possible to solve only one part of the domain, as stipulated on the control volume, depicted in FIGURE 1A. This simplification is performed with the purpose to reduce the computational effort.

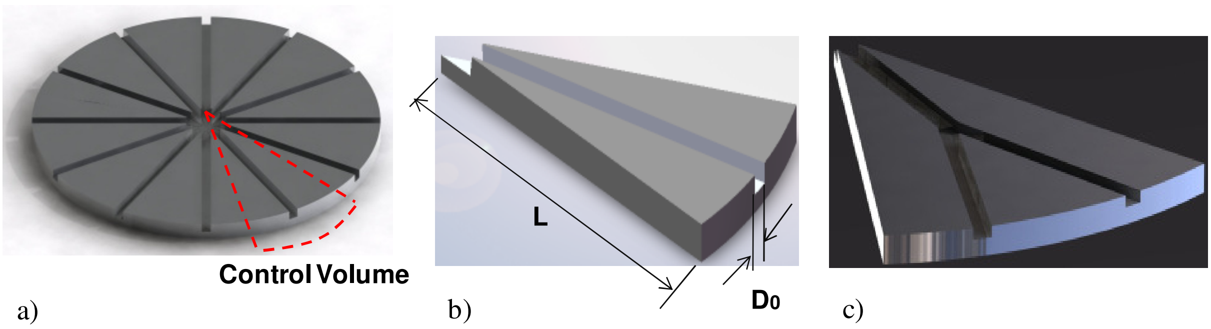


Figure 1 - Geometry configuration of the body with ducts: (a) global domain with control volume of the solved domain, (b) radial configuration, (c) bifurcated configuration.

As previously stated, in the present work the circular body is submitted to a constant heat flux in the lower surface (**q″** = 1 W/m²). Besides that, the thermophysical properties of the solid are kept constant, density of ρ_s = 55 kg/m³, specific heat of Cp_s = 1210 J/kgK and thermal conductivity of k_s = 0.027 W/mK. For remotion of heat from the solid body, fluid flows inside the ducts absorbing the energy of the body by forced convection. The thermophysical properties of the refrigerant fluid are also maintained constants, ρ_f = 0.995 kg/m³, Cp_f = 1009 J/kgK, k_f = 0.03 W/mK and the dynamical viscosity of the fluid of μ_f = 2.083 × 10^{-5} kg/ms. Concerning the boundary conditions, all surfaces are adiabatic, with exception of the lower surface that is heated with a constant flux. The fluid movement is imposed by a difference of pressure in the inlet and outlet of the domain, which are p_{inlet} = 0.062 Pa and p_{outlet} = 0 Pa, respectively.

The purpose of the present analysis is to determine the optimal geometry that is characterized by the minimum global thermal resistance $(T_{max} - T_{min})/(q'''A)$. The ratio between the ducts volume and the body volume is kept constant, being the global constraint, while the number of ducts varies for the radial configuration and the angle between the branches of ducts in the first construct, as well as, the ratio between the length of the single duct (L_0) and the radius of the body (L) vary for the bifurcated configuration.

The conservation equations of mass, momentum in *x*, *y* and *z* directions and energy for modelling incompressible flows at the steady state are, respectively, given by Bejan (1994):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_f}\frac{\partial P}{\partial x}+\upsilon\nabla^2 u \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho_f}\frac{\partial P}{\partial y}+\upsilon\nabla^2 v \tag{3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho_f}\frac{\partial P}{\partial w}+\upsilon\nabla^2 w \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_f\nabla^2 T \tag{5}$$

where *x*, *y* and *z* are the Cartesian coordinates (m), *u*, *v* and *w* are the velocity components in the *x*, *y* and *z* directions (m/s), ρ_f is the specific mass of the fluid (kg/m³), *u* is the kinematic viscosity of the fluid (m²/s), α_f is the thermal diffusivity of the fluid (m²/s), *T* is the temperature (°C or K), and ∇^2 means $\nabla^2=\partial/\partial x^2+\partial/\partial y^2$.

For the solid wall the conduction equation for the steady state and without heat generation is given by:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0 \tag{6}$$

For solving Eq. (1)-(6) numerically, the dimensionless coordinates, global thermal resistance, pressure, thermal conductivity, specific heat and density are obtained from Eq. (7) – (12):

$$\tilde{x},\tilde{y},\tilde{z},\tilde{D}_0,\tilde{L}_0,\tilde{t}=\frac{x,y,z,D_0,L_0,t}{L} \tag{7}$$

$$\tilde{T}=\frac{T-T_{\min}}{q''L/k_f} \tag{8}$$

$$\tilde{P}=\frac{P-P_{ref}}{\Delta P} \tag{9}$$

$$\tilde{k}=\frac{k_s}{k_f} \tag{10}$$

$$\tilde{C}_p=\frac{C_{ps}}{C_{pf}} \tag{11}$$

$$\tilde{\rho}=\frac{\rho_s}{\rho_f} \tag{12}$$

where P_{ref} is the pressure of the fluid on the outlet of ducts, T_{min} is the minimal temperature of the fluid on the inlet of the ducts, k_s and k_f are the thermal conductivities of the solid and the fluid, respectively, Cp_s and Cp_f are the specific heats of the solid and the fluid, respectively, ρ_s and ρ_f are the densities of the solid and the fluid, respectively, D_0 is the side of the square section of the main duct, L_0 is the length of the single duct for the bifurcated configuration, L is radius of the circular domain and t is the thickness of the circular body.

For brevity the dimensionless governing equations for the fluid flow and the solid domain will not be shown here.

The dimensionless parameters that govern the fluid flow with convection heat transfer along the duct are the Bejan (Be) and Prandtl (Pr) numbers, which are given, respectively, by:

$$Be = \frac{\Delta P L^2}{\alpha} \quad (13)$$

$$\Pr = \frac{\upsilon}{\alpha} \quad (14)$$

3. NUMERICAL MODEL

The function defined by Eq. (8) can be numerically determined by the solution of the Eq. (1) – (6) for the velocity in *x*, *y* and *z* directions, pressure and temperature on the solid and fluid domain. The system is solved as function of the degrees of freedom, as previously explained. The equation system, given by Eq. (1) – (6), is solved using a CFD package based on tetrahedral finite volume (FLUENT®, 2007). The solver is pressure based (coupled 1st order for pressure and upwind scheme for momentum and energy). The convergence was established when the residual are lower than 10^{-3} for the conservation of mass and momentum and lower than 10^{-6} for the energy equation. The code was previously validated by Rocha et al. (2009) and, for the sake of simplicity, will not be shown a new validation in the present work. The appropriate mesh was determined by successive refinements until the criterion, $|(T_{max,j} - T_{max,j+1})/ T_{max,j}| \leq 5.0 \times 10^{-4}$ was satisfied, where $T_{max,j}$ is the maximum temperature estimated using the current mesh and $T_{max,j+1}$ refers to the calculation which corresponds to the next mesh. Table 1 illustrates how grid independence test was performed for the radial configuration with n = 10 ducts intruded in the solid wall and for a fixed ratio between the volume of the ducts and the volume of the three-dimensional circular body, $\phi = 0.1$.

Table 1 – Numerical tests showing the achievement of grid independent for the radial configuration with $n = 10$ and $\phi = 0.1$ – criterion = 5.0×10^{-4}.

Number of Elements	T_{max}	$\lvert (T^{j}_{max} - T^{j+1}_{max})/T^{j}_{max} \rvert$
35036	310.4063	7.52E-04
241537	310.6397	4.41E-04
589630	310.7767	----------------

4. RESULTS

Firstly, it is investigated the thermal behavior of the system as function of the number of ducts inserted into a three-dimensional body for the radial configuration. To perform this investigation the ratio between the volume of the ducts and the volume of the body is kept fixed ($\phi = 0.1$). The Bejan and Prandtl numbers are also maintained fixed (Be = 10^8 and Pr = 0.71), for a laminar flow. Additionally, for all simulated cases of radial and bifurcated configuration the dimensionless thickness of the circular body are kept fixed, $\tilde{t} = 0.1$. The degree of freedom in this case was the ducts number, which was varied in the following range, $10 \le n \le 130$.

FIGURE 2 presents the global thermal resistance as function of the number of ducts intruded into the solid body. It is observed that the minimal number of ducts represents the worst global thermal resistance. As the number of ducts increases the global thermal resistance decreases until one specific number of ducts, i.e., achieves one optimal configuration, which in this case is $n = 46$. After this number of ducts the global thermal resistance increases again, leading to geometries of worse performance than for that with $n = 46$. The optimal global thermal resistance for this case is $\tilde{T} = 0.1516$. Additionally, the global thermal resistance for the best geometry (optimal) is approximately 77% lower than the one for the worst situation, i.e., the results showed that the employment of Constructal Design increases significantly the performance of the system.

For the case where the ducts are bifurcated, it is investigated the influence of the angle between the branches of the ducts over the global thermal resistance. FIGURE 3 presents the dimensionless global thermal resistance as function of the angle between the branches of the ducts for the following parameters (Be = 10^8, Pr = 0.71, ϕ = 0.1, $\tilde{L}_0$ = 0.2 and n = 12). It is also observed in this case one optimal geometric configuration. The optimal global thermal resistance and the angle between the branches are $\tilde{T}$ = 0.1742 and δ = 17.5º, respectively. Moreover, the optimal geometry led to a global thermal resistance approximately 40% lower than for the worst configuration. This result corroborates the fact that Constructal Design led to the best performance of the system, whatever the geometry studied.

FIGURE 4 presents the temperature topologies on the lower and upper surfaces for $\tilde{L}_0$ = 0.2, Figure. 4A and 4D, $\tilde{L}_0$ = 0.3, FIGURE 4B and 4E and for $\tilde{L}_0$ = 0.5, FIGURE 4C and 4F. It is remarkable that all topologies presented represent the optimal configuration as function of the angles between the branches of ducts (δ). On the one hand, for $\tilde{L}_0$ = 0.2, which represents the lowest distance from the center to the bifurcation point, the cooling is more efficient in the center of the body. However, for the periphery region, the temperature is higher than that obtained from the other configurations, i.e., the global thermal resistance is higher than for other configurations. In this case, the temperature field has not the best distribution in the radial direction. On the other hand, for $\tilde{L}_0$ = 0.5, which represents higher distance from the bifurcation to the center of the body, the temperature field is not the most distributed in the angular direction. For this configuration the optimal performance is not also achieved. The optimal geometry as function of $\tilde{L}_0$ is obtained for $\tilde{L}_0$ = 0.3, which represents the most uniform temperature distribution in both angular and radial directions. This result agrees with the findings of Bejan (2000) who states that a finite dimension system remains alive on time if it evolves to a geometric configuration that facilitates the current access through this system. In the present situation the current is the heat flux, which reaches the best configuration when the heat flux has it access facilitated, i.e., for the most well distributed temperature field.

FIGURE 5 presents the dimensionless global thermal resistance as function of the angle between branches (δ) for the bifurcated configuration for several values of $\tilde{L}_0$ = 0.2, $\tilde{L}_0$ = 0.3, $\tilde{L}_0$ = 0.4 and $\tilde{L}_0$ = 0.5, the other parameters are kept fixed. For all values of $\tilde{L}_0$ it was noticed points of local minimum global thermal resistance, where this system was optimized for the first time. Additionally, it was observed that optimal configuration occurs for $\tilde{L}_0$ = 0.3. However, $\tilde{L}_0$ = 0.3 did not represents the best performance for all range of δ, for instance, for the range 15 ≤ δ ≤ 17.5, $\tilde{L}_0$ = 0.2 performs lower global thermal resistance than for $\tilde{L}_0$ = 0.3.

FIGURE 6 presents the optimal global thermal resistance and the optimal angle between the branches of the ducts as function of $\tilde{L}_0$. The results showed that $\tilde{L}_0$ = 0.3 led to the best performance of the system, among the local optimal thermal resistance. The presented geometries had previously been optimized as function of the angle δ. Thus, the optimization as function of $\tilde{L}_0$ represents the second optimization, due to this fact the optimal global thermal resistance is presented as $(\tilde{T}_{opt})_{opt}$. The optimal angle of the branches increases as $\tilde{L}_0$ is increased, the optimal angle for $\tilde{L}_0$ = 0.3, which is the best geometry among all studied ones, is δ_{opt} = 20º.

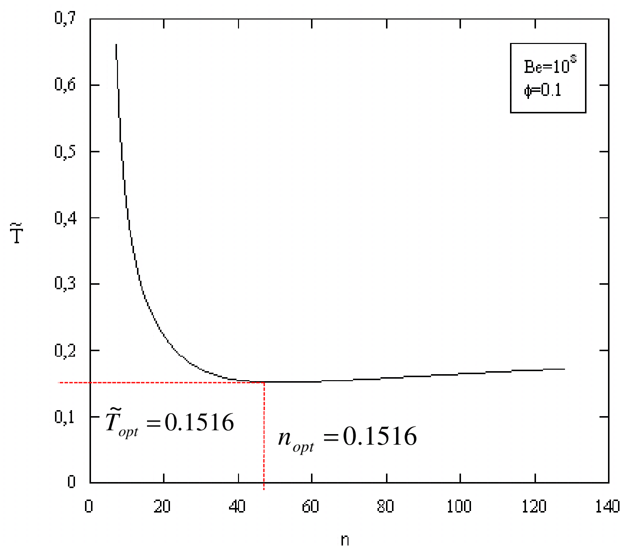


Figure 2 - Dimensionless global thermal resistance as function of the number of ducts for the radial configuration (ϕ = 0.1, Be = 10^8).

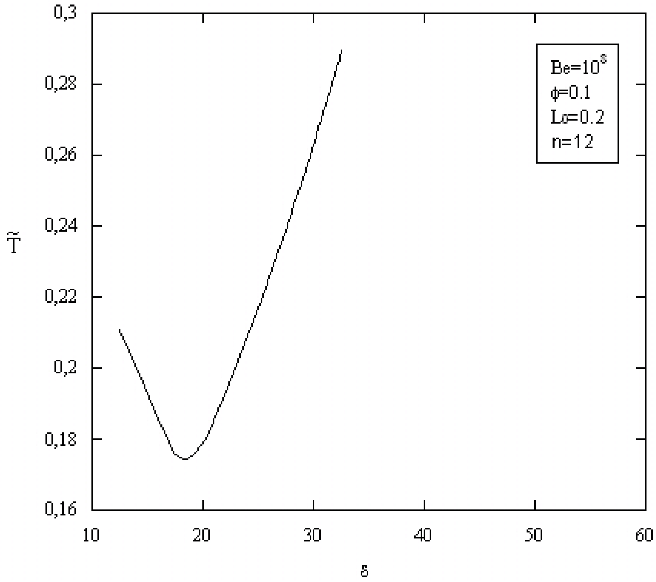
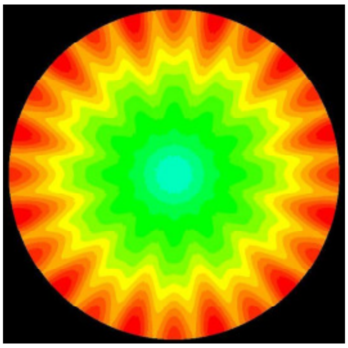
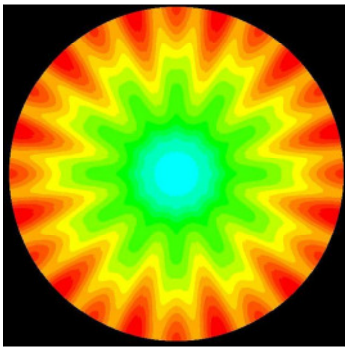


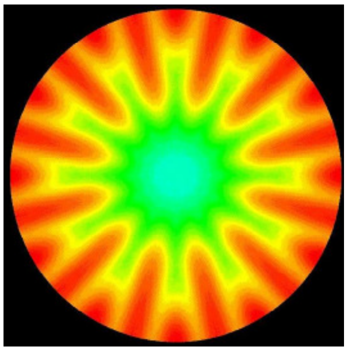
Figure 3 - Dimensionless global thermal resistance as function of the angle between the branches of the ducts for the bifurcated configuration ($\phi = 0.1$, Be = 10^8).



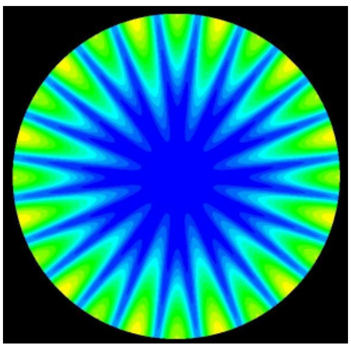
a)



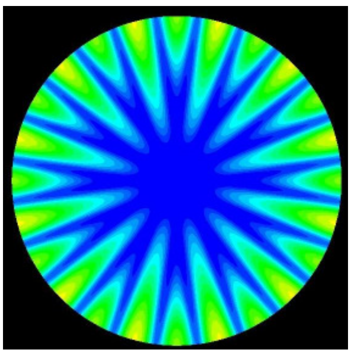
b)



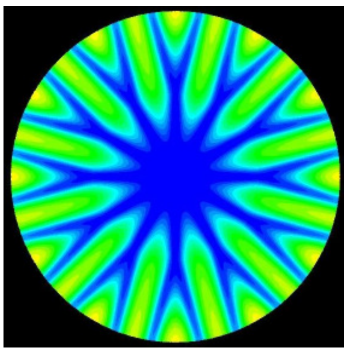
c)



d)



e)



f)

Figure 4 - Temperature distribution on the body as function of $\tilde{L}_0$: lower surface: (a) $\tilde{L}_0$ = 0.2, (b) $\tilde{L}_0$ = 0.3, (c) $\tilde{L}_0$ = 0.5, upper surface: (d) $\tilde{L}_0$ = 0.2, (e) $\tilde{L}_0$ = 0.3, (f) $\tilde{L}_0$ = 0.5.

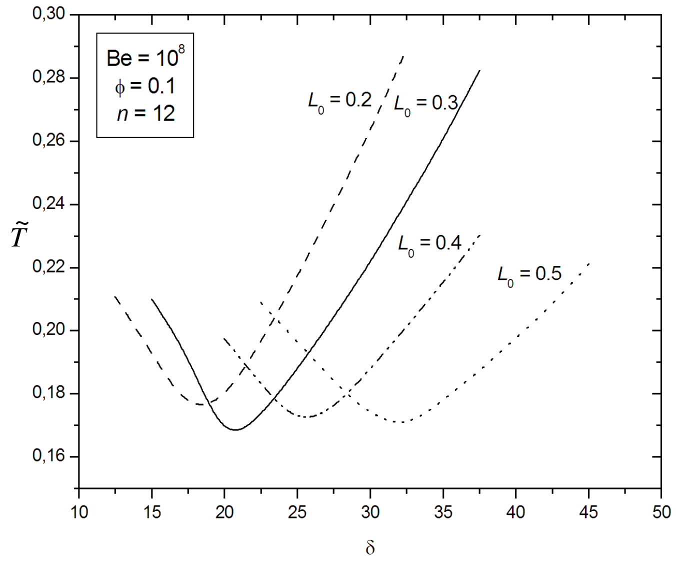


Figure 5 - Dimensionless global thermal resistance as function of the angle between branches of the ducts for the bifurcated configuration for several $\tilde{L}_0$ = 0.2, 0.3, 0.4 and 0.5 (ϕ = 0.1, Be = 10^8).

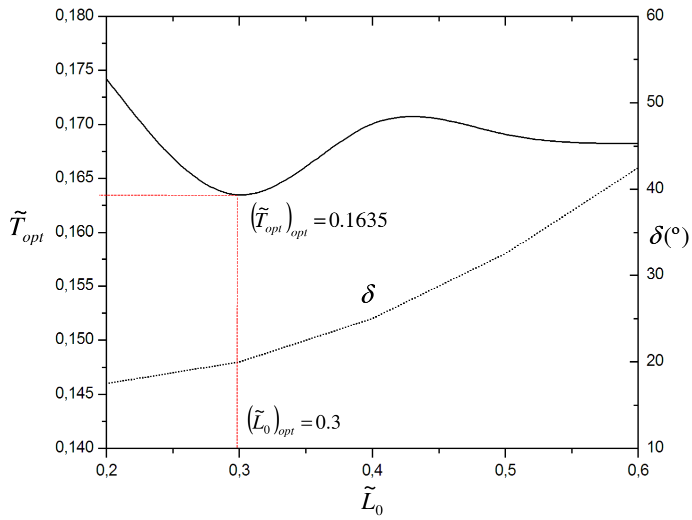


Figure 6 - Optimal Global thermal resistance and the optimal angle between the branches of the ducts as function of $\tilde{L}_0$.

One comparison between the radial and bifurcated configuration indicates that, for *n* = 12, the bifurcated configuration led to better results than the one obtained from the radial configuration. The global thermal resistance for best bifurcated configuration ($\tilde{L}_0$ = 0.3 and δ = 20º) was approximately 57% lower than for the radial configuration. This fact show the adaptation of the geometrical configuration in order to achieve the best performance and to remains alive.

5. CONCLUSIONS

In the present work was performed a numerical study for the optimization of the geometry of a three-dimensional body submitted to an intense heat flux and cooled by micro-channels. To achieve this goal it was employed Constructal Design. Two configurations were evaluated: ducts disposed in a radial form and ducts with one level of bifurcation. For the optimization the body, the channels volumes are kept constants, while the geometrical configuration varies. The degrees of freedom of the system were the number of ducts (n) for the radial configuration, and the angle between the branches (δ) and the ratio between the length of the single branch (L_0) and the radius of domain (L).

For the radial configuration there was one optimal configuration that is achieved when the number of ducts inserted into the solid body was n = 46. Additionally, the global thermal resistance for the best geometry (optimal) was approximately 77% lower than the one for the worst situation.

For the bifurcated configuration, it was investigated the influence of the angle between the branches of the ducts over the global thermal resistance. It was also observed that there is one optimal angle that led the system to one optimal configuration. For instance, the optimal geometry for $\tilde{L}_0$ = 0.2, FIGURE 3, led to a global thermal resistance approximately 40% lower than for the worst angle.

The influence of the ratio between the length of the single duct and the total length of the ducts over the global thermal resistance was also investigated. The temperature topologies indicates that the best performance of the system was achieved when the temperature field has the most uniform distribution in both angular and radial directions, which is in agreement with the findings of Bejan (2000).

The bifurcated system can be optimized twice by the employment of Constructal Design, for the two degrees of freedom evaluated: δ and $\tilde{L}_0$.

One comparison between the radial and bifurcated configuration indicates that, for n = 12, the bifurcated configuration led to better results than the one obtained from the radial configuration. The global thermal resistance for best bifurcated configuration ($\tilde{L}_0$ = 0.3 and δ = 20º) was approximately 57% lower than for the radial configuration.

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