Development of a Bio-inspired Hybrid Decomposition Algorithm Based on Whale and Differential Evolution Strategies for Multiobjective Optimization☆

Desenvolvimento de um Algoritmo de Decomposição Híbrido Bioinspirado Baseado em Baleias e Estratégias de Evolução Diferencial para Otimização Multiobjetivo

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Abstract

A Multiobjective Optimization Problem (MOP) requires the optimization of several objective functions simultaneously, usually in conflict with each other. One of the most efficient algorithms for solving MOPs is MOEA/D (Multiobjective Evolutionary Algorithm Based on Decomposition), which decomposes a MOP into single-objective optimization subproblems and solves them using information from neighboring subproblems. MOEA/D variants with other evolutionary operators have emerged over the years, improving their efficiency in various MOPs. Recently, the IWOA (Improved Whale Optimization Algorithm) was proposed, an optimization algorithm bioinspired by the whale hunting method hybridized with Differential Evolution, which presented excellent results in single-objective optimization problems. This work proposes the MOEA/D-IWOA algorithm, which associates characteristics of the evolutionary operators of the IWOA to MOEA/D. Computational experiments were accomplished to analyze the performance of the MOEA/D-IWOA in benchmark MOPs suites. The results were compared with those obtained by the MOEA/D, Non-dominated Sorting Genetic Algorithm II (NSGA-II), Third Evolution Step of Generalized Differential Evolution (GDE3), Improving the Strength Pareto Evolutionary Algorithm (SPEA2), and Indicator-Based Evolutionary Algorithm (IBEA) algorithms in the Hypervolume and Inverted Generational Distance Plus (IGD+) indicators. The MOEA/D-IWOA proved to be competitive, with a good performance profile, in addition to presenting the best results in some POMs.

Keywords

Multiobjective Optimization • MOEA/D • IWOA

Resumo

Um Problema de Otimização Multiobjetivo (POM) requer a otimização de várias funções objetivo simultaneamente, geralmente conflitantes entre si. Um dos algoritmos mais eficientes para resolver POMs é o MOEA/D (*Multiobjective Evolutionary Algorithm Based on Decomposition*), que decompõe um POM em subproblemas de otimização monobjetivo, isto é, com uma única função objetivo a ser minimizada, e os resolve usando informações de subproblemas vizinhos. Variantes do MOEA/D com outros operadores evolutivos surgiram ao longo dos anos, melhorando sua eficiência

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em diversos POMs. Recentemente foi proposto o IWOA (*Improved Whale Optimization Algorithm*), um algoritmo de otimização bioinspirado no método de caça das baleias hibridizado com Evolução Diferencial que apresentou ótimos resultados em problemas de otimização monobjetivo. Esse trabalho propõe o algoritmo MOEA/D-IWOA, que extende o IWOA para resolver POMs associando características dos seus operadores evolutivos ao MOEA/D. Experimentos computacionais para analisar o desempenho do MOEA/D-IWOA em POMs *benchmark* foram realizados e os resultados comparados aos obtidos pelos algoritmos bem conhecidos da literatura, a saber, MOEA/D, *Non-dominated Sorting Genetic Algorithm II* (NSGA-II), *Third Evolution Step of Generalized Differential Evolution* (GDE3), *Improving the Strength Pareto Evolutionary Algorithm* (SPEA2) e *Indicator-Based Evolutionary Algorithm* (IBEA) nos indicadores Hypervolume e *Inverted Generational Distance Plus* (IGD+). O MOEA/D-IWOA se mostrou competitivo, com bom perfil de desempenho, além de apresentar os melhores resultados em alguns POMs.

Palavras-chave

Otimização Multiobjetivo • MOEA/D • IWOA

1 Introduction

Many problems originating in real world can be modeled as a Multiobjective Optimization Problem (MOP), which means a situation that requires minimization and/or maximization of several objective functions simultaneously, usually conflicting with each other. This means that improving one objective can degrade another. Solving a MOP means getting a set of solutions that present the best compromises between the objectives, known as Pareto optimal solutions.

Multiobjective Evolutionary Algorithms (MOEAs) are among the most popular methods to solve a MOP since a single execution can find a set of Pareto optimal solutions, and the decision maker chooses the one that attends the preferences. Multiobjective Evolutionary Algorithm Based on Decomposition (MOEA/D) [1] is one of the most efficient MOEAs to solve a MOP. The algorithm decomposes the MOP into several single-objective subproblems simultaneously solved using aggregation functions. Each individual from the population represents the current best solution to one of the subproblems, and each subproblem is solved using information from neighboring subproblems.

MOEA/D variants appeared in literature and improved their efficiency in several MOPs. A survey of the works involving MOEA/D presented a comprehensive view of the main components' improvements [2]. One of the aspects pointed out by the authors was the change in MOEA/D's evolutionary operators.

Li and Zhang [3] proposes a new MOEA/D based on Differential Evolution (DE) [4] called MOEA/D-DE, which outperforms the well-know Non-dominated Sorting Genetic Algorithm II (NSGA-II) [5] in MOPs with complicated Pareto Fronts. Ke et al. [6] used Ant Colonies Optimization (ACO) and proposed the MOEA/D-ACO algorithm, which presented better performance than MOEA/D with conventional genetics operators and local search in all nine test instances of 0-1 multiobjective knapsack problem. Martínez and Coello [7] used Particle Swarm Optimization (PSO) in the algorithm named multiobjective decomposition-based PSO algorithm (dMOPSO). This algorithm outperformed the MOEA/D and another MOEA based on PSO in most of the test problems adopted by the authors.

Recently, Mirjalili and Lewis [8] proposed an optimization algorithm called WOA (*Whale Optimization Algorithm*), inspired by the whale hunting method that generates a bubbles curtain around its prey, keeping them close to the surface and making the final attack easier. WOA has proven to be competitive with other existing algorithms, as can be seen in a literature review of WOA research [9]. In some cases, the WOA algorithm may show premature convergence, which makes it get stuck in local optima. To overcome this limitation, Bozorgi and Yazda [10] proposes IWOA (*Improved* WOA), where WOA is hybridized with DE, which has an excellent capacity to explore the search space. The experimental results presented in [10] were performed on 25 single-objective optimization problems and showed that IWOA could improve the performance of WOA in these problems.

This work proposes the MOEA/D-IWOA algorithm, which extends IWOA to MOPs using the MOEA/D structure. Numerical experiments with benchmark suites ZDT [11], DTLZ [12], and WFG [13] problems were performed. The performance indicators Hypervolume [14] and Inverted Generational Distance Plus (IGD+) [15] were adopted to evaluate the MOEA/D-IWOA's performance. The obtained results were compared to the classic MOEAs from the literature: NSGA-II, Third Evolution Step of Generalized Differential Evolution (GDE3) [16], Indicator-Based Evolutionary Algorithm (IBEA) [17], and Improving the Strength Pareto Evolutionary Algorithm (SPEA2) [18], in addition to the original MOEA/D itself.

The rest of the work is organized as follows: Section 2 describes the methods used in this work, while Section 3 shows the numerical experiments carried out. Discussions and an analysis of the obtained results are presented in Section 4. Finally, Section 5 presents the conclusions.

2 Methods

2.1 MOPs

The MOPs treated in this work with m objective functions can be written as:

$$\begin{array}{ll} \min & F(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \dots, f_m(\boldsymbol{x})) \\ s.a. & x_i \in (l_i, u_i) \quad \forall i = 1, \dots, n \\ & \boldsymbol{x} = (x_1, \dots, x_n) \in \mathbb{R}^n \end{array} \tag{1}$$

Given $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, we say that $\boldsymbol{x} \prec \boldsymbol{y}$ (read as $\boldsymbol{x}$ dominates $\boldsymbol{y}$) if $f_i(\boldsymbol{x}) \leq f_i(\boldsymbol{y})$, $\forall i = 1, \dots, n$, and there is some j integer between 1 and n such that $f_j(\boldsymbol{x}) < f_j(\boldsymbol{y})$. If $\boldsymbol{x} \nprec \boldsymbol{y}$ and $\boldsymbol{y} \nprec \boldsymbol{x}$, $\boldsymbol{x}$ and $\boldsymbol{y}$ are said to be non-dominated by each other [19]. The set of Pareto optimal solutions is formed by non-dominated solutions that are not dominated by any other.

2.2 MOEA/D

MOEA/D works by decomposing a MOP (Eq. (1)) into several single-objective optimization subproblems and optimizing them simultaneously. Consider $\boldsymbol{\lambda}^1, \dots, \boldsymbol{\lambda}^N$ a set of weight vectors and $\boldsymbol{z}^* = (z_i, \dots, z_m)$ a reference point, where z_i is the best value found so far for the objective function f_i. Using the Tchebycheff aggregate function, the objective function of the jth problem can be defined as

$$g(\boldsymbol{x}|\boldsymbol{\lambda}^j, \boldsymbol{z}^*) = \max\{\lambda_i^j |f_i(\boldsymbol{x}) - z_i^*|\} \tag{2}$$

in which $\boldsymbol{\lambda}^j = (\lambda_1^j, \dots, \lambda_m^j)$, where $\lambda_i \geq 0$ with $i = 1, \dots, m$ and $\sum_{i=1}^m \lambda_i = 1$.

For each $\boldsymbol{\lambda}^j$, among the other weight vectors, those closest are considered its neighborhood. This way, the neighborhood of the jth subproblem will be defined by the subproblems that have their weight vector in the neighborhood of $\boldsymbol{\lambda}^j$. Thus, a population is formed with the best solution found for each subproblem (Eq. (2)), which will be used for the rest of the algorithm (reproduction and updating of solutions). A complete description of MOEA/D can be found in [1].

2.3 IWOA

Both WOA and IWOA were initially designed to solve single-objective optimization problems. In WOA, a randomly created candidate solutions population behaves like whales looking for food that surround their prey using a bubble net and move in a spiral until they concentrate their prey and attack. In this way, WOA is divided into three actions to evolve each component x_i of the solution $\boldsymbol{x}$: search (Eq. (3)), surround (Eq. (4)), and attack (Eq. (5)):

$$\begin{array}{c} D = \left|Cx_{rand_i} - x_i\right| \\ x_i = x_{rand_i} - AD, \end{array} \tag{3}$$

where x_{rand_i} is the ith component of $\boldsymbol{x}_{rand}$ (a randomly chosen solution), $A = 2ar - a$, $C = 2r$, a decreases linearly from 2 to 0 and r is a random number in $[0, 1]$;

$$\begin{array}{c} D = \left|Cx_i^* - x_i\right| \\ x_i = x_i^* - AD, \end{array} \tag{4}$$

and

$$\begin{array}{c} x_i = \begin{cases} x_i^* - AD, & \text{if } p < 0.5 \\ D' e^{bl} \cos(2\pi l) + x_i^*, & \text{if } p \geq 0.5 \end{cases} \\ D' = \left|x_i^* - x_i\right|, \end{array} \tag{5}$$

where x_i^* is the ith component of $\boldsymbol{x}^*$ (the best solution obtained so far), b is a constant to define the shape of the logarithmic spiral, l is a random number in $[-1, 1]$, and p is a random number between [0,1].

In the search action, WOA updates the position of each whale according to a randomly chosen whale $\boldsymbol{x}_{rand}$ (Eq. (3)). In the surround action, WOA updates the position of each whale according to the best-positioned whale $\boldsymbol{x}^*$ (Eq. (4)), that is, the one that represents the best objective function value obtained so far. Finally, the whales swim in attacking, decreasing encirclement and spiraling simultaneously (Eq. (5)). There is a 50% probability that the WOA will update each whale's position by surround or attack actions, representing the refinement of the best solution obtained so far, known as exploitation phase.

In IWOA, WOA is hybridized with DE, which has good search space exploration capability. This way, it combines this feature of DE with the exploitation phase of WOA, providing promising solutions. IWOA uses the following DE mutation: for each $\boldsymbol{x} \in \mathbb{R}^n$ of the N candidate solutions, two solutions $\boldsymbol{x}_{r1}$ and $\boldsymbol{x}_{r2} \in \mathbb{R}^n$ are randomly selected from the population, different from each other and $\boldsymbol{x}$. The following formula calculates $\boldsymbol{v} \in \mathbb{R}^n$ [4]:

$$\boldsymbol{v} = \boldsymbol{x}^* + F(\boldsymbol{x}_{r1} - \boldsymbol{x}_{r2}), \tag{6}$$

where $F \in \mathbb{R}$ is a perturbation rate. The vector $\boldsymbol{u}$ is found mixing $\boldsymbol{x}$ and $\boldsymbol{v}$. For each component, a random number $rand_i$ in $[0, 1]$ is chosen. If $rand_i \leq CR$ (user-defined crossover rate), then $u_i = v_i$, otherwise $u_i = x_i$. More details about the IWOA can be found at [10].

2.4 The Proposed MOEA/D-IWOA

The MOEA/D-IWOA algorithm, which extends IWOA to MOPs using the MOEA/D structure, adopted the swarm intelligence mechanism of IWOA as evolutionary operators for MOEA/D. They drive the algorithm to generate promising offspring once it is an effective way to create new solutions for a MOP. Algorithm 1 shows the pseudo-code for MOEA/D-IWOA.

generate $\boldsymbol{\lambda}^1, \dots, \boldsymbol{\lambda}^N$: weight vectors distributed uniformly
for $i \neq j \leq N$ **do** ▷ N : population size
  calculate Euclidean distance between $\boldsymbol{\lambda}^i$ and $\boldsymbol{\lambda}^j$;
end for
for $i \leq N$ **do**
  set neighborhood of i as $B(i) = \{i_1, \dots, i_T\}$, where $\boldsymbol{\lambda}^{i_1}, \dots, \boldsymbol{\lambda}^{i_T}$ are the T closest weight vectors to $\boldsymbol{\lambda}^i$
end for
generate initial population $\{\boldsymbol{x}_1, \dots, \boldsymbol{x}_N\}$ randomly;
calculate objective function $F(\boldsymbol{x}_i) = (f_1(\boldsymbol{x}_i), \dots, f_m(\boldsymbol{x}_i))^T$ of each individual $\boldsymbol{x}_i$ in the initial population;
initialize $\boldsymbol{z} = (z_1, \dots, z_m)^T$, where $z_i = \min\{f_i(\boldsymbol{x}_1), \dots, f_i(\boldsymbol{x}_N)\}$
for $1 \leq g \leq G$ **do** ▷ g: current generation and G: maximum generation
  for $i \leq N$ **do**
    select two index k and l from $B(i)$ and calculate $\boldsymbol{v}$ using Eq. (6).
    for $1 \leq j \leq n$ **do** ▷ n : dimention of $\boldsymbol{x}_i$
      if $rand \leq 1 - g/G$ **then** ▷ $rand$: a random number in $[0, 1]$
        if $rand \leq CR$ **then**
          $\boldsymbol{Offspring}(j) = \boldsymbol{v}(j)$
        else
          $\boldsymbol{Offspring}(j) = \boldsymbol{x}_i(j)$, where $\boldsymbol{x}_i$ is the whale position after using Eq. (3).
        end if
      else
        if $rand \leq 0.5$ **then**
          $\boldsymbol{Offspring}(j) = \boldsymbol{x}_i(j)$, where $\boldsymbol{x}_i$ is the whale position after using Eq. (4).
        else
          $\boldsymbol{Offspring}(j) = \boldsymbol{x}_i(j)$, where $\boldsymbol{x}_i$ is the whale position after using Eq. (5).
        end if
      end if
    end for
    update $\boldsymbol{z}$
    calculates Tchebycheff aggregate function (Eq. (2)) for $\boldsymbol{Offspring}$ and call it by g_1
    for $k \in B(i)$ **do**
      calculates Tchebycheff aggregate function for $\boldsymbol{x}_k$ and call it by g_2
      if $g_1 < g_2$ **then**
        $\boldsymbol{x}_k = \boldsymbol{Offspring}$
      end if
    end for
  end for
end for

Algorithm 1: MOEA/D-IWOA.

3 Numerical Experiments

3.1 Performance Indicators

The performance indicators Hypervolume and Inverted Generational Distance Plus (IGD+) are adopted here as mappings that assign scores to Pareto front approximations.

Given a point set $X \subset \mathbb{R}^d$ and a reference point $\boldsymbol{r} \in \mathbb{R}^d$, the Hypervolume indicator is

$$H(X) = \lambda\left(\bigcup_{\boldsymbol{p}\in X}[\boldsymbol{p},\boldsymbol{r}]\right) \quad (7)$$

where $[\boldsymbol{p},\boldsymbol{r}] = \{\boldsymbol{q} \in \mathbb{R}^d \mid \boldsymbol{p} \prec \boldsymbol{q} \wedge \boldsymbol{q} \prec \boldsymbol{r}\}$ and $\lambda(\cdot)$ denotes the Lebesgue measure. Hypervolume was introduced as a tool for analyzing multiobjective optimization algorithms by [14]. It assesses the optimization process results by taking into account multiple aspects, such as the proximity of the solutions to the Pareto front, diversity, and spread (Fig. 1).

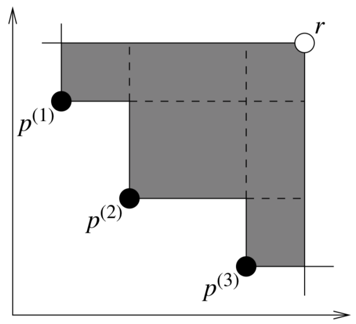

Figure 1: Illustration of a two objective example with Hypervolume indicator (area shown in grey), where $\boldsymbol{p}^{(1)}$, $\boldsymbol{p}^{(2)}$, and $\boldsymbol{p}^{(3)}$ form the set X of obtained Pareto optimal solutions and $\boldsymbol{r}$ a reference point - Extract from [20].

Denoting the cardinality of a set Z by $|Z|$, the Inverted Generational Distance (IGD) indicator is defined as

$$\mathrm{IGD}(A) = \frac{1}{|Z|}\left(\sum_{j=1}^{|Z|} \hat{d}_j^p\right)^{1/p} \quad (8)$$

where $\hat{d}_j$ is the Euclidean distance from $\boldsymbol{z}_j$ to its nearest objective vector in A. The IGD Plus (IGD+) is the IGD indicator with the follow modified distance calculation:

$$d^+(\boldsymbol{z},\boldsymbol{a}) = \sqrt{d_1^{+2} + \cdots + d_m^{+2}} = \sqrt{(\max\{a_1 - z_i, 0\})^2 + \cdots + (\max\{a_m - z_m, 0\})^2}. \quad (9)$$

Note that the higher the Hypervolume value, better the MOEA performance. In IGD+, the lower the value, better the performance of the MOEA.

Performance profiles [21] serve as a way to visualize and comprehend the outcomes of experiments. Let $t_{p,s}$ be the performance indicator obtained by the algorithm s in problem p. The performance ratio can be defined as

$$r_{p,s} = \frac{t_{p,s}}{\min\{t_{p,s} : s \in S\}}. \quad (10)$$

The performance profile $\rho_s(\tau)$ of and algorithm s can be defined as

$$\rho_s(\tau) = \frac{1}{n_p}\left|\{p \in P : r_{p,s} \leq \tau\}\right|. \quad (11)$$

That is, $\rho_s(\tau)$ is the probability that the performance ratio $r_{p,s}$ of algorithm $s \in S$ is within a factor $\tau \geq 1$ of the best possible ratio. The area under the performance profile curve serves as a overall performance indicator for s in the set of problems P. The greater the area, the more efficient the algorithm is.

3.2 Results

The benchmark problem suites adopted in the numerical experiments of this work were ZDT, DTLZ, and WFG, which Pareto fronts obtained were evaluated according to performance indicators Hypervolume and IGD+. In addition to ZDT problems, which have two objective functions, the DTLZ and WFG problems considered were set with the usual configurations, considering three objective functions. The results obtained by the MOEA/D-IWOA were compared to those of the classic MOEAs from the literature: NSGA-II, GDE3, IBEA, SPEA2, and the original MOEA/D itself. All MOEAs were run 20 times with a population of 100 candidate solutions and 20000 evaluations of each objective function. In addition, the Wilcoxon rank sum non-parametric statistical hypothesis test was performed to verify significant differences between the samples. The true Pareto front of each problem for calculating the performance indicator is determined by selecting all non-dominated solutions from all algorithms' combined sets of results.

To improve the distribution of solutions obtained on the MOP's Pareto Front, the MOEA/D-IWOA considered the neighborhood size of each subproblem as 2, up to half of the objective function evaluations. From then on, it considered ten until the end of the algorithm execution. The MOEA/D-IWOA employed the DE and WOA parameters of $F = 0.5$, $CR = 0.9$, and $b = 1$ (Eq. 5) in its computational experiments. On the other hand, the remaining MOEAs utilized the parameters as specified in their respective original papers. Tables 1, 2, and 3 show the means and standard deviations of the results obtained by the analyzed algorithms. The best results are in bold. Figures 3 and 2 show the performance profile curves, and Table 4 presents the areas under them.

Table 1: Results obtained considering different strategies for the ZDT functions. STD is standard deviation.

		MOEA/D-IWOA	MOEA/D	NSGA-II	GDE3	IBEA	SPEA2
				ZDT-1			
Hypervolume	Mean	**0.7201**	0.7161	0.7183	0.6077	0.7200	0.7186
	STD	0.0001	0.0047	0.0004	0.0241	0.0001	0.0004
IGD+	Mean	**0.0025**	0.0048	0.0039	0.0834	0.0027	0.0037
	STD	0.0000	0.0027	0.0003	0.0182	0.0000	0.0003
				ZDT-2			
Hypervolume	Mean	**0.4444**	0.4077	0.4426	0.2533	0.4439	0.4429
	STD	0.0002	0.1084	0.0004	0.0373	0.0001	0.0005
IGD+	Mean	**0.0026**	0.0360	0.0038	0.1455	0.0032	0.0035
	STD	0.0001	0.1017	0.0002	0.0329	0.0001	0.0003
				ZDT-3			
Hypervolume	Mean	0.5992	**0.6296**	0.6034	0.5569	0.5970	0.5989
	STD	0.0002	0.0456	0.0198	0.0201	0.0051	0.0002
IGD+	Mean	**0.0021**	0.0113	0.0034	0.0815	0.0028	0.0024
	STD	0.0000	0.0111	0.0049	0.0215	0.0031	0.0002
				ZDT-4			
Hypervolume	Mean	0.3352	0.6919	**0.7127**	0.0000	0.6373	0.7123
	STD	0.2035	0.0118	0.0039	0.0000	0.0501	0.0042
IGD+	Mean	0.3558	0.0216	**0.0074**	2.2579	0.0725	0.0080
	STD	0.2117	0.0088	0.0025	0.7277	0.0495	0.0031
				ZDT-6			
Hypervolume	Mean	**0.3889**	0.3856	0.3853	0.3874	0.3873	0.3842
	STD	0.0000	0.0083	0.0014	0.0014	0.0004	0.0029
IGD+	Mean	**0.0020**	0.0039	0.0045	0.0032	0.0031	0.0052
	STD	0.0000	0.0050	0.0010	0.0010	0.0002	0.0021

Table 2: Results obtained considering different strategies for the DTLZ functions. STD is standard deviation.

		MOEA/D-IWOA	MOEA/D	NSGA-II	GDE3	IBEA	SPEA2
			DTLZ-1				
Hypervolume	Mean	0.8050	0.8017	0.7428	0.3722	0.4888	**0.8128**
	STD	0.0124	0.0045	0.2039	0.3464	0.0666	0.0958
IGD+	Mean	0.0254	**0.0224**	0.0522	0.3595	0.1175	0.0242
	STD	0.0053	0.0012	0.0859	0.4680	0.0251	0.0282
			DTLZ-2				
Hypervolume	Mean	0.5354	0.5271	0.5311	0.5331	**0.5575**	0.5550
	STD	0.0017	0.0010	0.0028	0.0048	0.0012	0.0011
IGD+	Mean	0.0335	0.0363	0.0346	0.0331	0.0262	**0.0260**
	STD	0.0006	0.0004	0.0012	0.0017	0.0008	0.0006
			DTLZ-3				
Hypervolume	Mean	0.1298	**0.1371**	0.0462	0.0000	0.0371	0.0550
	STD	0.2121	0.2058	0.1223	0.0000	0.0597	0.1374
IGD+	Mean	1.1503	0.9938	1.7103	11.3662	**0.7559**	1.4272
	STD	1.0069	0.8114	1.1401	4.2699	0.4988	0.8496
			DTLZ-4				
Hypervolume	Mean	0.4559	0.2130	0.5327	0.5295	**0.5578**	0.4917
	STD	0.1249	0.1442	0.0043	0.0108	0.0010	0.0971
IGD+	Mean	0.1152	0.3783	0.0341	0.0350	**0.0260**	0.0867
	STD	0.1312	0.1655	0.0015	0.0045	0.0007	0.0935
			DTLZ-5				
Hypervolume	Mean	0.1955	0.1948	0.1990	**0.1997**	0.1985	0.1994
	STD	0.0002	0.0000	0.0002	0.0001	0.0004	0.0002
IGD+	Mean	0.0053	0.0057	0.0029	**0.0023**	0.0031	**0.0023**
	STD	0.0001	0.0000	0.0002	0.0001	0.0002	0.0001
			DTLZ-6				
Hypervolume	Mean	0.1958	0.1948	0.1994	**0.2003**	0.1962	0.2001
	STD	0.0004	0.0000	0.0002	0.0000	0.0011	0.0000
IGD+	Mean	0.0052	0.0057	0.0024	**0.0018**	0.0050	**0.0018**
	STD	0.0003	0.0000	0.0001	0.0000	0.0007	0.0000

Table 3: Results obtained considering different strategies for the WFG functions. STD is standard deviation.

		MOEA/D-IWOA	MOEA/D	NSGA-II	GDE3	IBEA	SPEA2
			WFG-1				
Hypervolume	Mean	0.7656	0.8960	0.8839	0.2017	**0.9305**	0.8754
	STD	0.0277	0.0343	0.0243	0.0602	0.0064	0.0286
IGD+	Mean	0.3729	0.1195	0.1917	1.4498	**0.0700**	0.1898
	STD	0.0541	0.0502	0.0403	0.1574	0.0134	0.0493
			WFG-2				
Hypervolume	Mean	0.9061	0.8947	0.9149	0.8928	**0.9284**	0.9248
	STD	0.0042	0.0062	0.0033	0.0090	0.0015	0.0027
IGD+	Mean	0.0658	0.0787	0.1149	0.1387	**0.0342**	0.0790
	STD	0.0029	0.0034	0.0128	0.0187	0.0013	0.0062
			WFG-3				
Hypervolume	Mean	0.3953	0.4020	0.3918	0.3473	**0.4084**	0.3776
	STD	0.0022	0.0030	0.0046	0.0116	0.0031	0.0061
IGD+	Mean	0.0602	0.0449	0.0810	0.1817	**0.0269**	0.0986
	STD	0.0055	0.0071	0.0158	0.0294	0.0046	0.0131
			WFG-4				
Hypervolume	Mean	0.5283	0.5247	0.5141	0.5082	**0.5533**	0.5334
	STD	0.0017	0.0045	0.0059	0.0060	0.0017	0.0049
IGD+	Mean	0.1187	0.1227	0.1424	0.1475	**0.0947**	0.1170
	STD	0.0025	0.0036	0.0079	0.0071	0.0025	0.0068
			WFG-5				
Hypervolume	Mean	0.4822	0.4752	0.4873	0.4884	**0.5139**	0.5074
	STD	0.0029	0.0031	0.0044	0.0026	0.0035	0.0051
IGD+	Mean	0.1701	0.1795	0.1699	0.1657	**0.1450**	0.1488
	STD	0.0023	0.0020	0.0033	0.0035	0.0021	0.0039
			WFG-6				
Hypervolume	Mean	0.4900	0.4596	0.4646	0.4728	**0.4996**	0.4878
	STD	0.0131	0.0164	0.0124	0.0140	0.0169	0.0154
IGD+	Mean	0.1763	0.2110	0.2155	0.2053	**0.1695**	0.1837
	STD	0.0194	0.0239	0.0192	0.0219	0.0244	0.0234
			WFG-7				
Hypervolume	Mean	0.5266	0.5246	0.5181	0.4701	**0.5561**	0.5403
	STD	0.0027	0.0030	0.0044	0.0138	0.0009	0.0026
IGD+	Mean	0.1258	0.1232	0.1370	0.2144	**0.0912**	0.1083
	STD	0.0037	0.0019	0.0062	0.0231	0.0015	0.0037
			WFG-8				
Hypervolume	Mean	0.4443	0.4469	0.4360	0.4286	**0.4788**	0.4523
	STD	0.0024	0.0023	0.0043	0.0057	0.0017	0.0028
IGD+	Mean	0.2474	0.2378	0.2717	0.2830	**0.2088**	0.2468
	STD	0.0032	0.0027	0.0080	0.0085	0.0025	0.0046
			WFG-9				
Hypervolume	Mean	0.4781	0.4777	0.4930	0.4389	**0.5326**	0.5024
	STD	0.0347	0.0340	0.0074	0.0429	0.0028	0.0250
IGD+	Mean	0.1781	0.1801	0.1609	0.2488	**0.1103**	0.1470
	STD	0.0554	0.0540	0.0107	0.0703	0.0026	0.0408

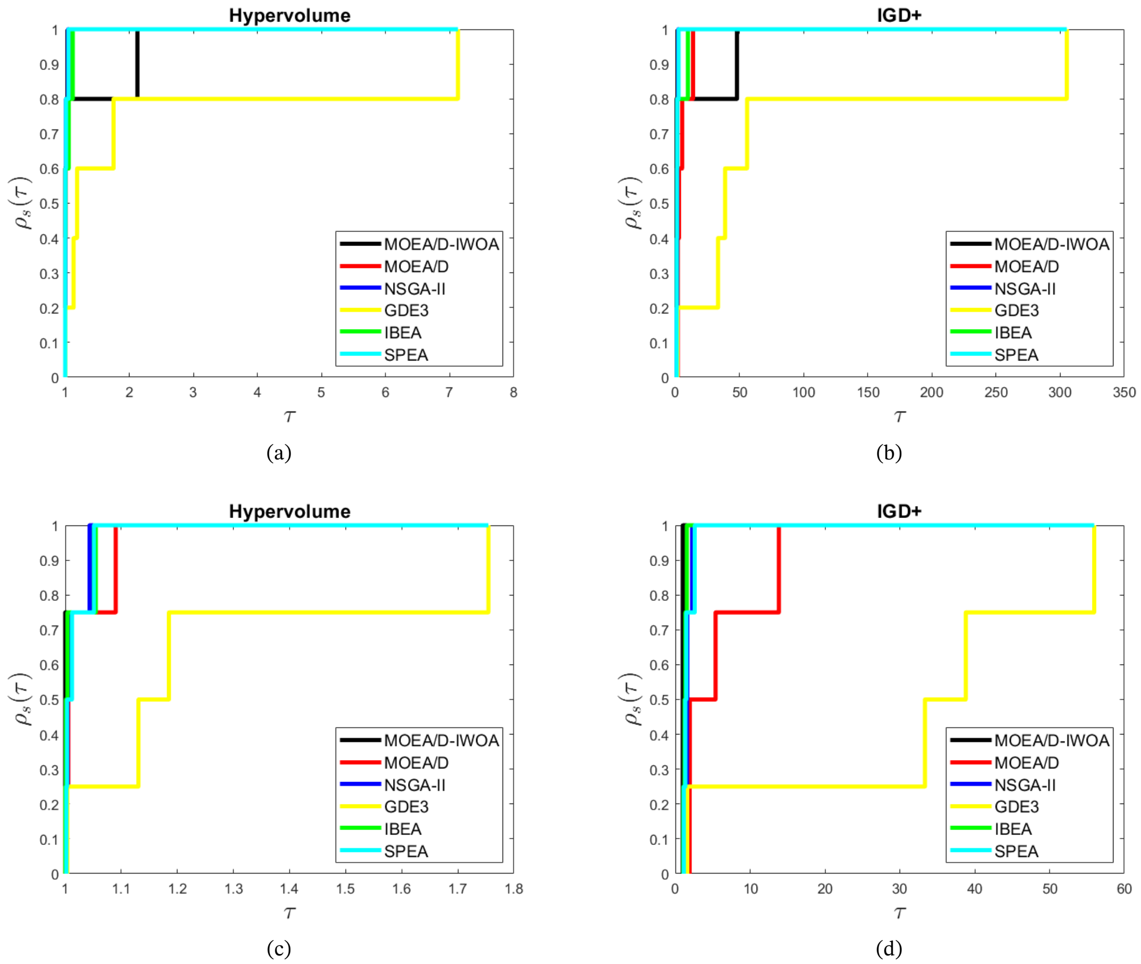

Figure 2: Performance profile curves of ZDT suite problems: (a) and (b) refer to all the ZDT problems, and (c) and (d) exclude the ZDT-4 problem.

Table 4: Areas under Performance Profiles Curves.

	MOEA/D-IWOA	MOEA/D	NSGA-II	GDE3	IBEA	SPEA2
ZDT						
Hypervolume	0.9634	0.9976	**1.0000**	0.7664	0.9961	0.9997
IGD+	0.9707	0.9879	0.9998	0.7186	0.9952	**1.0000**
ZDT without ZDT-4 problem						
Hypervolume	**1.0000**	0.9820	0.9971	0.6552	0.9969	0.9939
IGD+	**1.0000**	0.9131	0.9869	0.4281	0.9946	0.9883
DTLZ						
Hypervolume	**1.0000**	0.9820	0.9764	0.8205	0.9603	0.9834
IGD+	0.9510	0.8321	0.9939	0.6965	0.9646	**1.0000**
WFG						
Hypervolume	0.9799	0.9827	0.9825	0.8604	**1.0000**	0.9886
IGD+	0.9541	0.9742	0.9531	0.8174	**1.0000**	0.9620
Total of ones						
	3	0	1	0	2	2

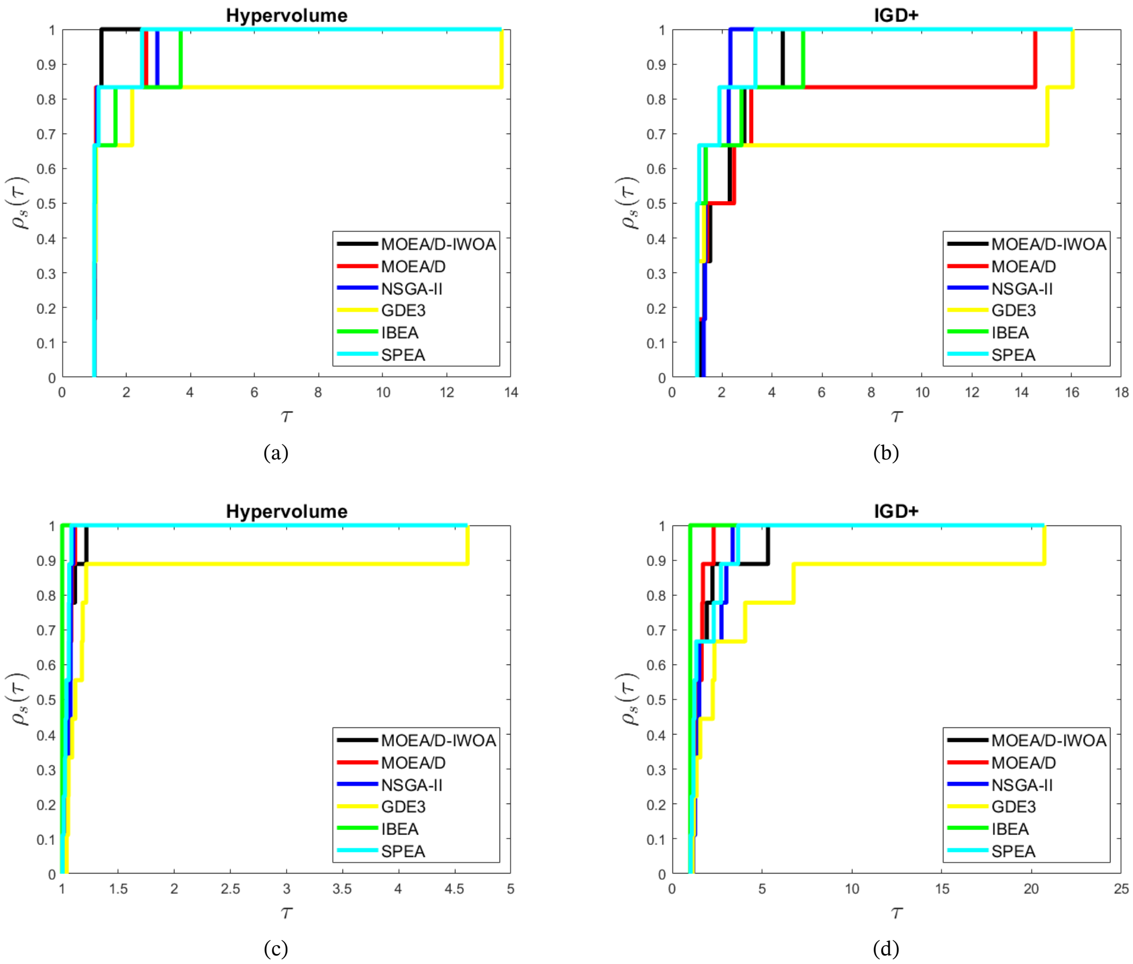

Figure 3: Performance profile curves: (a) and (b) refer to DTLZ problems, and (c) and (d) refer to WFG problems.

4 Results Analysis and Discussion

In problems ZDT-1, ZDT-2 and ZDT-6, the MOEA/D-IWOA showed better performance compared to other MOEAs both in Hypervolume and in IGD+, all with statistically significant differences according to Wilcoxon's non-parametric test (p-value ≤ 0.05). It also obtained the best performance in the ZDT-3 problem considering only the IGD+ results. Still in ZDT-3, despite presenting a lower Hypervolume mean than MOEA/D and NSGA-II, there were no statistically significant differences according to Wilcoxon's non-parametric test (p-value > 0.05). In the ZDT-4 problem, the MOEA/D-IWOA failed to perform well in both indicators, presenting a better result only when compared to GDE3. Due to this, the area under the performance profile curves was not the highest (Fig. 2(a-b) and Tab. 4). However, excluding ZDT-4 problem, MOEA/D-IWOA shows the largest area under the performance profile curves, indicating that it gets the best overall performance in the ZDT suite (Fig. 2(c-d) and Tab. 4). The ZDT4 problem involves many local optimal Pareto frontiers. This means that MOEA/D-IWOA did not perform well in this kind of problem. However, it outperforms the other algorithms in the rest of the suite.

In DTLZ problems, MOEA/D-IWOA stood out mainly in DTLZ-1 and DTLZ-3, in which it obtained the second best Hypervolume. In addition, in none of the DTLZ problems, there statistically significant differences according to Wilcoxon's non-parametric test concerning the best MOEA in the Hypervolume and IGD+. Even though it was not the best Hypervolume in any DTLZ problem, MOEA/D-IWOA had the best Hypervolume overall performance (Tab. 4), as it was close to the best in all problems (Tab. 2).

In WFG problems, the main highlight of MOEA/D-IWOA was in problem WFG-6, where it obtained the second best Hypervolume and IGD+. Although MOEA/D-IWOA did not have an overall outstanding performance in WFG suite, none of the WGF problems there were statistically significant differences according to the Wilcoxon's non-parametric test in relation to the best MOEA in the Hypervolume and IGD+.

Table 4 shows that the MOEA/D-IWOA algorithm demonstrated competitiveness in general, as it had the largest area under the curve in three of the analyzed groups. IBEA and SPEA2 followed close behind, with the biggest area under the curve in two groups, while NSGA-II held the top spot in only one group. The original MOEA/D did not perform better in any of the evaluated groups. This way, using IWOA in evolutionary operators within MOEA/D proved to be a promising variant capable of improving its performance in several MOPs.

Due to its mathematical formulation, the Tchebycheff aggregate function is widely used in MOEA/D and can result in evenly distributed weight vectors across the Pareto front, leading to positive outcomes in various problem scenarios. Nevertheless, it's important to consider other aggregation functions to achieve even better results [22].

5 Conclusions

In this work, the MOEA/D-IWOA algorithm, an IWOA extension for MOPs using the MOEA/D framework, is proposed. The proposed MOEA/D-IWOA algorithm was used to solve benchmark MOPs from the ZDT, DTLZ and WFG suites. The performance of the MOEA/D-IWOA was compared with that of the MOEAs NSGA-II, GDE3, SPEA2, MOEA/D and IBEA in the indicators Hypervolume and IGD+. In general, the results showed that the MOEA/D-IWOA had the best performance in some problems, many with statistically significant differences. Even when it didn't get the best result, it was competitive with the best, often without statistically significant differences between its results. Furthermore, MOEA/D-IWOA was superior to the original MOEA/D algorithm, showing that using IWOA in evolutionary operators within MOEA/D is a promising variant capable of improving its performance. Future works may include MOEA/D-IWOA analysis in MOPs with more than three objective functions, called Many-objective optimization problems, in addition to coupled IWOA in other MOEAs, IBEA for example, to evaluate if its performance in WFG problems can be improved. In addition, to evaluate engineering system design with constraints.

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References

[1] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 6, pp. 712–731, 2007. Available at: https://doi.org/10.1109/TEVC.2007.892759

[2] Q. Xu, Z. Xu, and T. Ma, "A survey of multiobjective evolutionary algorithms based on decomposition: Variants, challenges and future directions," *IEEE Access*, vol. 8, pp. 41 588–41 614, 2020. Available at: https://doi.org/10.1109/ACCESS.2020.2973670

[3] H. Li and Q. Zhang, "Multiobjective optimization problems with complicated Pareto sets, MOEA/D and NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 2, pp. 284–302, 2009. Available at: https://doi.org/10.1109/TEVC.2008.925798

[4] R. Storn and K. Price, "Differential Evolution – a simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, vol. 11, no. 4, pp. 341–359, 1997. Available at: https://doi.org/10.1023/A:1008202821328

[5] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002. Available at: https://doi.org/10.1109/4235.996017

[6] L. Ke, Q. Zhang, and R. Battiti, "MOEA/D-ACO: A multiobjective evolutionary algorithm using decomposition and Ant Colony," *IEEE Transactions on Cybernetics*, vol. 43, no. 6, pp. 1845–1859, 2013. Available at: https://doi.org/10.1109/TSMCB.2012.2231860

[7] S. Z. Martínez and C. A. C. Coello, "A multi-objective particle swarm optimizer based on decomposition," in *Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation GECCO '11*. New York, NY, USA: Association for Computing Machinery, 2011, p. 69–76. Available at: https://doi.org/10.1145/2001576.2001587

[8] S. Mirjalili and A. Lewis, "The Whale Optimization Algorithm," *Advances in Engineering Software*, vol. 95, pp. 51–67, 2016. Available at: https://doi.org/10.1016/j.advengsoft.2016.01.008

[9] F. S. Gharehchopogh and H. Gholizadeh, "A comprehensive survey: Whale optimization algorithm and its applications," *Swarm and Evolutionary Computation*, vol. 48, pp. 1–24, 2019. Available at: https://doi.org/10.1016/j.swevo.2019.03.004

[10] S. M. Bozorgi and S. Yazdani, "IWOA: An improved whale optimization algorithm for optimization problems," *Journal of Computational Design and Engineering*, vol. 6, no. 3, pp. 243–259, 2019. Available at: https://doi.org/10.1016/j.jcde.2019.02.002

[11] E. Zitzler, K. Deb, and L. Thiele, "Comparison of multiobjective evolutionary algorithms: Empirical results," *Evolutionary Computation*, vol. 8, no. 2, pp. 173–195, 2000. Available at: https://doi.org/10.1162/106365600568202

[12] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable multi-objective optimization test problems," in *Proceedings of the 2002 Congress on Evolutionary Computation. CEC'02*, vol. 1, 2002, pp. 825–830. Available at: https://doi.org/10.1109/CEC.2002.1007032

[13] S. Huband, L. Barone, L. While, and P. Hingston, "A scalable multi-objective test problem toolkit," in *Evolutionary Multi-Criterion Optimization*, C. A. Coello Coello, A. Hernández Aguirre, and E. Zitzler, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2005, pp. 280–295. Available at: https://doi.org/10.1007/978-3-540-31880-4_20

[14] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach," *IEEE Transactions on Evolutionary Computation*, vol. 3, no. 4, pp. 257–271, 1999. Available at: https://doi.org/10.1109/4235.797969

[15] H. Ishibuchi, H. Masuda, Y. Tanigaki, and Y. Nojima, "Modified distance calculation in generational distance and inverted generational distance," in *Evolutionary Multi-Criterion Optimization*, A. Gaspar-Cunha, C. Henggeler Antunes, and C. C. Coello, Eds. Cham: Springer International Publishing, 2015, pp. 110–125. Available at: https://doi.org/10.1007/978-3-319-15892-1_8

[16] S. Kukkonen and J. Lampinen, "GDE3: the third evolution step of generalized differential evolution," in *2005 IEEE Congress on Evolutionary Computation*, vol. 1, 2005, pp. 443–450. Available at: https://doi.org/10.1109/CEC.2005.1554717

[17] E. Zitzler and S. Künzli, "Indicator-based selection in multiobjective search," in *Parallel Problem Solving from Nature - PPSN VIII*, X. Yao, E. K. Burke, J. A. Lozano, J. Smith, J. J. Merelo-Guervós, J. A. Bullinaria, J. E. Rowe, P. Tiňo, A. Kabán, and H.-P. Schwefel, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2004, pp. 832–842.

[18] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the strength Pareto evolutionary algorithm for multiobjective optimization," in *Evolutionary Methods for Design, Optimization and Control with Applications to Industrial Problems, EUROGEN 2001, Athens.*, vol. 3242, 2001.

[19] K. Deb and D. Kalyanmoy, *Multi-Objective Optimization Using Evolutionary Algorithms*. USA: John Wiley & Sons, Inc., 2001.

[20] C. Fonseca, L. Paquete, and M. Lopez-Ibanez, "An improved dimension-sweep algorithm for the hypervolume indicator," in *2006 IEEE International Conference on Evolutionary Computation*, 2006, pp. 1157–1163. Available at: https://doi.org/10.1109/CEC.2006.1688440

[21] E. D. Dolan and J. J. Moré, "Benchmarking optimization software with performance profiles," *Mathematical Programming*, vol. 91, pp. 201–213, 2002. Available at: https://doi.org/10.1007/s101070100263

[22] X. Ma, Q. Zhang, G. Tian, J. Yang, and Z. Zhu, "On Tchebycheff decomposition approaches for multiobjective evolutionary optimization," *IEEE Transactions on Evolutionary Computation*, vol. 22, no. 2, pp. 226–244, 2018. Available at: https://doi.org/10.1109/TEVC.2017.2704118