

# Finite Element Analysis of a Spur Gear Considering Mass Relief Strategies<sup>☆</sup>

## Análise de Elementos Finitos de uma Engrenagem Cilíndrica de Dentes Retos Considerando Métodos de Alívio de Massa

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### Abstract

This paper deals with analyzing the structural influence of mass reliefs in spur gears. For this purpose, a system composed of pinion and a gear was designed, such that for gear several geometries were designed with different reliefs shapes and soul thicknesses. From the proposed geometries, finite element analysis (FEA) was performed, and the tooth stresses of each model were compared with the solid gear. From the results, it was observed that the tooth stresses are reduced in some cases. Besides, from the aforementioned cases, it is possible to observe that the maximum stresses may take place in its core instead of the teeth (rim area). On the other hand, based on other cases, the core thickness plays an important role as a criterion that defines the local stress.

### Keywords

Spur Gears • Finite Element Analysis • Optimization • Computational Mechanics • Machine Elements Design

### Resumo

Este artigo trata da análise da influência estrutural de alívios de massa em engrenagens. Para tanto, um sistema composto por um pinhão e uma engrenagem foi projetado e, para a engrenagem, foram projetadas várias geometrias com diferentes formatos de alívio e espessura de alma. A partir das geometrias propostas, análises de elementos finitos (FEA) foram realizadas, e as tensões nos dentes de cada modelo foram comparadas com às da engrenagem sólida. A partir dos resultados, foi observado que as tensões nos dentes diminuem em alguns casos. Além disso, a partir dos casos supracitados, é possível observar que as maiores tensões podem ocorrer na alma ao invés do dente (aro externo). Por outro lado, baseado nos casos observados, a espessura da alma é um critério relevante para definição da tensão nesse local.

### Palavras-chave

Engrenagens Cilíndricas de Dentes Retos • Análise de Elementos Finitos • Otimização • Mecânica Computacional • Projeto de Elementos de Máquinas

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## 1 Introduction

Gears are crucial parts of many mechanical transmission systems and, presently, increase the need for more optimized components that use less material and energy but can still meet performance requirements. The advancement and diffusion of CAD (Computer-Aided-Design) and CAE (Computer-Aided-Engineering) technologies allow the design of components to be increasingly optimized and complex, which could not be developed by traditional analytical methods. In addition, this approach minimizes the high costs of prototypes and physical tests, enabling, still in the design phase, failure prediction and improvements opportunities.

In this scenario, this paper is devoted to investigating the effect of mass relief on the structural behavior of the gear body due to the adoption of several strategies of mass reduction. The tool to be used is the Finite Element Analysis, where all results will be compared to a reference model composed of a solid gear. This paper did not aim to calculate the real stresses in the analyzed gears, being restricted to a comparison based on the maintenance of the mesh in the interest regions, as detailed in other sections.

## 2 Literature Review

Wilcox and Coleman [1] used finite elements to develop equations that could predict gear tooth stresses. The results for the new formulas were graphically compared with previous results that used photoelastic data. Usually, the curvatures in all cases were in good agreement, with the stress results of the new formulation being up to 20% lower than the others. Furthermore, they concluded that the observed differences might be more due to the difficulties encountered in the photoelastic technique than in the finite element method.

The stress distribution in the fillets of spur gears was investigated by Andrews [2] by using finite element analysis varying the point of force application, presenting a comparison with data from previously performed photoelastic tests. The results obtained showed a close correlation between the internal and external stresses of the gear tooth. Moreover, the results from this work were compared to photoelastic tests.

Filiz and Evercioglu [3] evaluated the stresses in spur gear teeth using parabolic triangular finite elements under three conditions. In the first, the contact force was applied on one point of the tooth [3]. For the second scenario, the force was distributed over a few points. Finally, the third simulation was performed simulating the teeth contact. It was possible to notice that when applying the force on one point, the results for the contact stress were significantly different from the distributed force and contact simulation. However, regarding the tooth bending stress, it was not noticed significant differences between the three conditions.

Hassan [4] analyzed the contact stresses on gear teeth for several contact angles between the teeth using quadratic quadrilateral iso-parametric plane stress finite elements[4]. The stress magnitude obtained from the simulations was compared with those calculated according to the AGMA standard, achieving similar results.

Using a plane finite element model, Gonçalves et al. [5] evaluated the influence of holes inserted in different positions of a spur gear. It was verified that when the holes are at a certain distance from the tooth root, there is no variation on the tooth stresses, regardless of the hole (or material avoided in case of spoked gears) position.

Prabhakaran et al. [6] compared the bending stresses in a spur gear obtained from Lewis equation, AGMA standards, and the finite element method, using solid elements. Analysis was performed for different gear modules, maintaining the same load condition. The results showed less than 4% difference between the finite element analysis and the AGMA standard and 12% between the finite element and Lewis equation results.

Balaji et al. [7] compared the contact stresses in spur gear calculated from Lewis equation, AGMA standards, and finite elements, modeling the gear tooth with solid elements. Analysis was performed for several gear modules and the results obtained attested to a good correlation between the results of the AGMA standard and the finite element method, using the Hertz formula to calculate the contact stress, with a difference of up to 2%.

## 3 Methodology

In order to perform the analysis, a set of pinion/gear with dimensions presented in Table 1 was designed. Del Mastro [8] presented a methodology to design gears with mass reliefs considering the main gear dimensions and engine parameters. Following this methodology with the input parameters presented in Table 2, gears with mass reliefs were designed, as illustrated in Figure 1.

The main dimensions of the design optimizations are presented in Table 3 and Figure 2. In order to analyze more optimization possibilities, it was designed more gears, maintaining the dimensions presented in Table 3, only reducing the gear core width to 4 mm. Altogether, eleven gear were designed and modeled in CAD platform, one

with no mass reliefs, five models with 6 mm of gear core width, and five with 4 mm. Just for information, the core is defined as a region of the gear that connects its hub to its rim.

Table 1: Typical gear dimensions.

Input	
Module (mm)	2.50
Pressure angle (deg)	20.00
Face width	30.00
Number of pinion teeth	20
Number of gear teeth	70
Pinion key height (mm)	8.00
Gear key height (mm)	10.00
Pinion shaft diameter (mm)	23.00
Gear shaft diameter (mm)	32.00
Output	
Gear pitch diameter (mm)	175.00
Gear circle base diameter (mm)	168.75
Gear outer diameter (mm)	180.00

Table 2: Engine parameters used as input data for the mass relief design.

Rotation (rpm)	1000
Power (hp)	14.95
Torque (Nm)	30.00

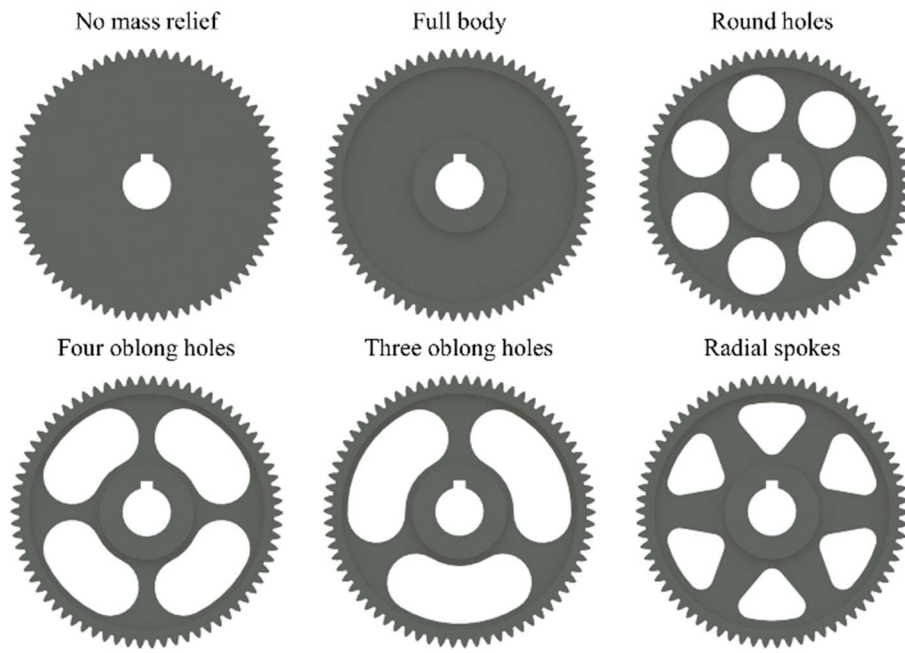


Figure 1: Types of gears designed to be evaluated.

Table 3: Main geometric parameters of the optimized gears according to Del Mastro [8].

$a$	Gear core width (mm)	6.0
$K_e$	Gear minimum material (mm)	6.0
$d_a$	Relief diameter (mm)	156.0
$d_c$	Hub diameter (mm)	63.0
$d_m$	Average diameter (mm)	110.0
$d_f$	Hole relief diameter (mm)	38.0
$n_{f0}$	Number of round holes	7
$2a$	Distance between four oblong holes (mm)	12.0
$2.7a$	Distance between three oblong holes (mm)	16.2
$n_f$	Number of radial spokes	6
$L$	Radial spoke width (mm)	22.0
$R$	Radial relief radius (mm)	8.0

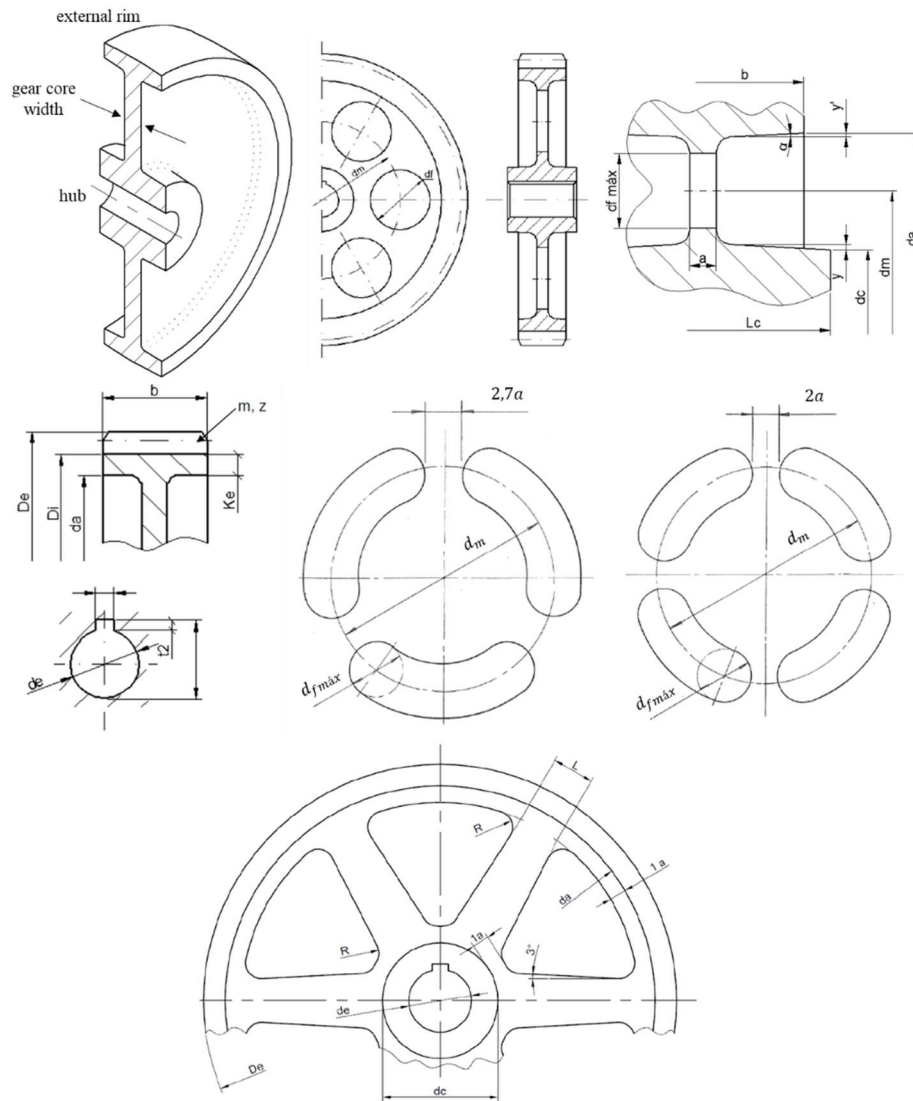


Figure 2: Main dimensions of the optimized gears.

The finite element analysis (FEA) pre-processing was made with HyperMesh<sup>®</sup> software for the OptiStruct<sup>®</sup> solver. The meshes were composed of hexahedral and pentagonal linear solid elements due to the gear's geometry. It was mapped using surface element and generated so that the gear outer rim elements of all eleven models were identical to reinforce the comparative character of the analysis, as can be verified in the figures given in Appendixes A and B. Furthermore, the mesh in the region of teeth in contact was refined to improve the accuracy of the results in this region. The steel properties available in the software were used, with Young's modulus of 210 GPa and a Poisson's ratio of 0.30.

The boundary conditions were applied through rigid elements. It was applied torque of 30 Nm on the pinion, the constraint of all Degree of Freedom (DOF) of the gear and five DOF of the pinion, leaving only the rotation about the z-axis free, as can be seen in Figure 3. All contacts between om teeth of the pinion and the gear were also configured, as shown in Figure 4, which characterizes a non-linear static analysis.

For the analysis, it was chosen to evaluate the equivalent von Mises stress since it takes into account all element stresses. It was also used a method to approximate the stress from the elements to their nodes on the post-processor by averaging the stress tensor and calculating the invariants from this one. To perform the computational work, a personal computer with main characteristics: Intel Processor i5 Dual-Core, 2.50 GHz 2.71 GHz and 8 GB RAM. For all numerical assessments, the processing time will be presented.

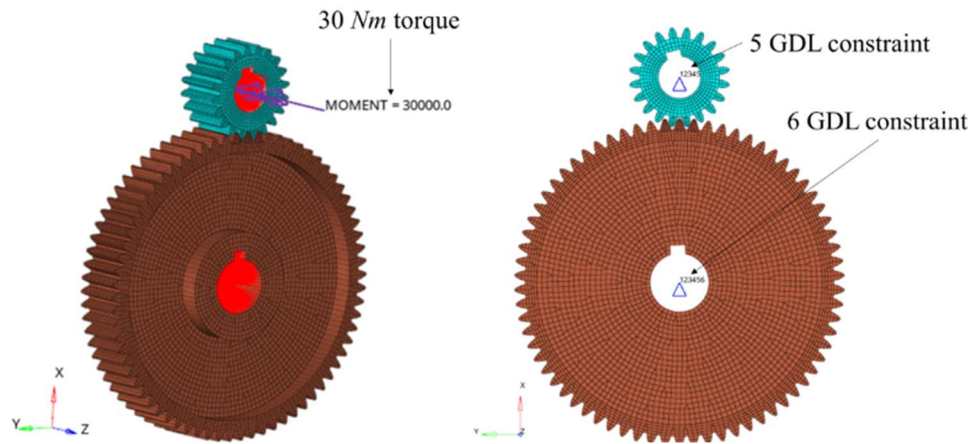


Figure 3: Boundary conditions.

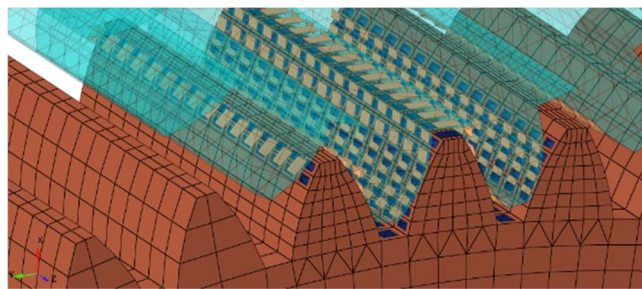


Figure 4: Gear contact setup.

## 4 Results

Table 4 presents the volumes of each geometry designed and its percentage variation with respect to the gear with no reliefs, taken as reference. The volumes, compared graphically in Figure 5, were measured by using the CAD tool. To facilitate the reference regarding each geometry, they have been identified with acronyms initiated by the capital letter G.

Table 4: Gear geometric concepts, geometric parameters and processing time.

Gear	Geometric Concept	Core Width (mm)	Volume (mm <sup>3</sup> )	Mass Reduction	Model DOF	Processing Time (s)
G1	No Mass Relief	-	692,706	-	223,152	566
G2	Full Body	4	308,797	-55.42%	156,960	73
G3		6	276,805	-60.04%	145,026	67
G4	Round Holes	4	261,165	-62.30%	151,224	69
G5		6	245,050	-64.62%	140,724	59
G6	4 Oblong Holes	4	248,584	-64.11%	145,920	66
G7		6	236,663	-65.84%	136,746	63
G8	3 Oblong Holes	4	246,836	-64.37%	144,600	63
G9		6	235,498	-66.00%	136,071	93
G10	Radial Spokes	4	270,816	-60.91%	148,776	71
G11		6	246,836	-64.37%	138,888	86

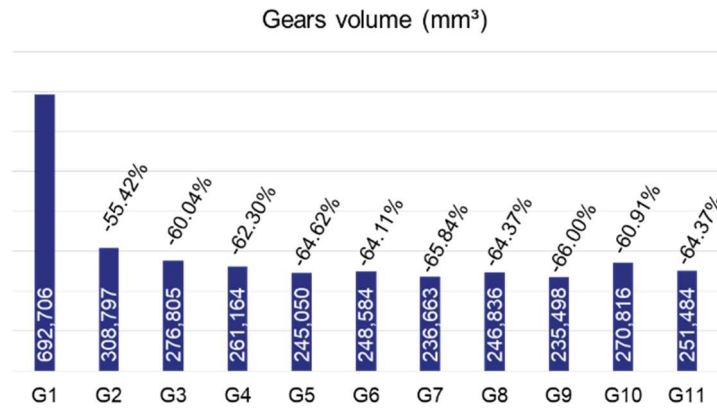


Figure 5: Gears volume.

As this work is only a comparative analysis between the various designed geometries and taking into account that the mesh in the region of interest is the same for all models, it was considered that it is not necessary to perform a mesh convergence test, since calculating the actual stresses is not the purpose of the present contribution.

The stress distribution of each model is given in Appendix C. The tooth stresses are presented in detail in Appendix D. The nodes for evaluation of the tooth stresses are presented in Figure 6. In Table 5 are the stress results of each simulation. A comparison between the average values of the contact stresses is presented in Figure 7, taking the gear with no mass reliefs as the reference, represented by the solid red line. Similarly, as can be seen in Figure 8, the bending stresses of each model are also compared.

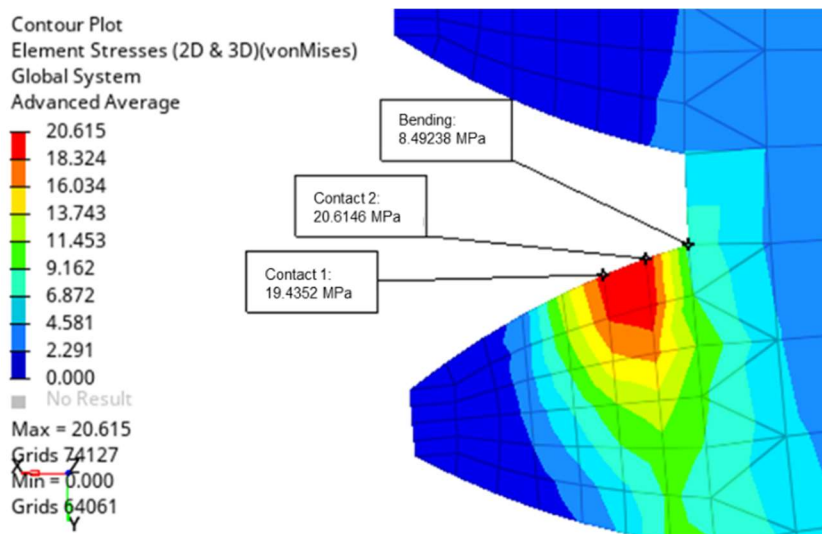


Figure 6: Stress evaluation nodes.



Table 5: Stress results of each gear and its variations.

Gear	Stress (MPa)				Stress variation		
	Contact 1	Contact 2	Bending	Gear core	Average contact	Bending	Maximum at gear core
G1	19.4352	20.6146	8.49238	-	-	-	-
G2	18.9906	19.6681	7.44564	-	-3.47%	-12.33%	-
G3	18.9067	19.5647	7.32904	-	-3.94%	-13.70%	-
G4	19.3469	18.887	5.70741	25.4410	-4.53%	-32.79%	-
G5	19.2607	18.582	5.3488	37.3693	-5.51%	-37.02%	46.89%
G6	19.5432	19.8669	7.03328	48.0957	-1.60%	-17.18%	-
G7	18.0541	19.6054	6.7994	69.7994	-5.97%	-19.94%	45.13%
G8	19.4251	18.984	7.16672	58.2586	-4.10%	-15.61%	-
G9	19.2992	19.553	6.89241	82.6492	-2.99%	-18.84%	41.87%
G10	19.1483	19.2455	6.51156	13.3450	-4.13%	-23.32%	-
G11	19.0720	19.0496	6.27353	19.3149	-4.81%	-26.13%	44.74%

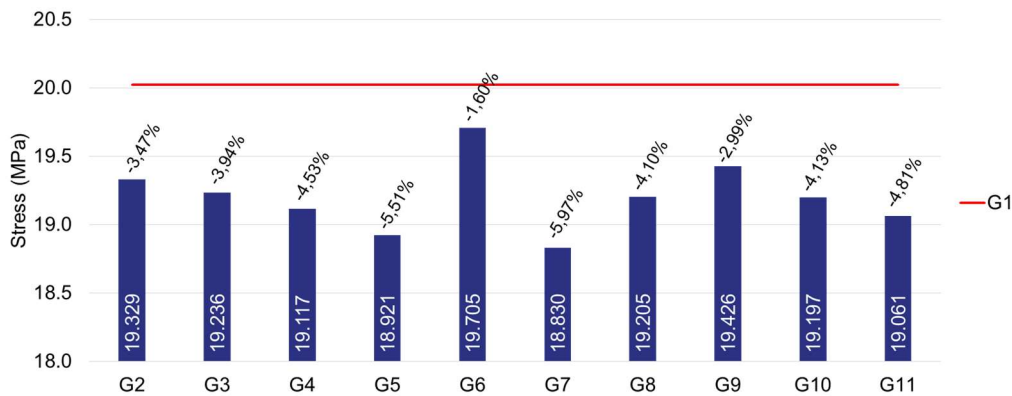


Figure 7: Tooth gear contact stress.



Figure 8: Tooth gears bending stress.

As can be noticed, the contact and bending stresses decreased in the models with mass reliefs. It can also be noticed that all the models with a gear core of 4 mm exhibited lower stresses than similar ones. Furthermore, during the analyses, it was noticed that in some models, the maximum stress occurs on the gear body and not on the tooth, diverging from what was verified in the reference gear. This result can be seen in Figure 9, where the results of gear G1 and gear G4 are shown. A comparison between the contact and core stresses of the gears is shown in Figure 10.



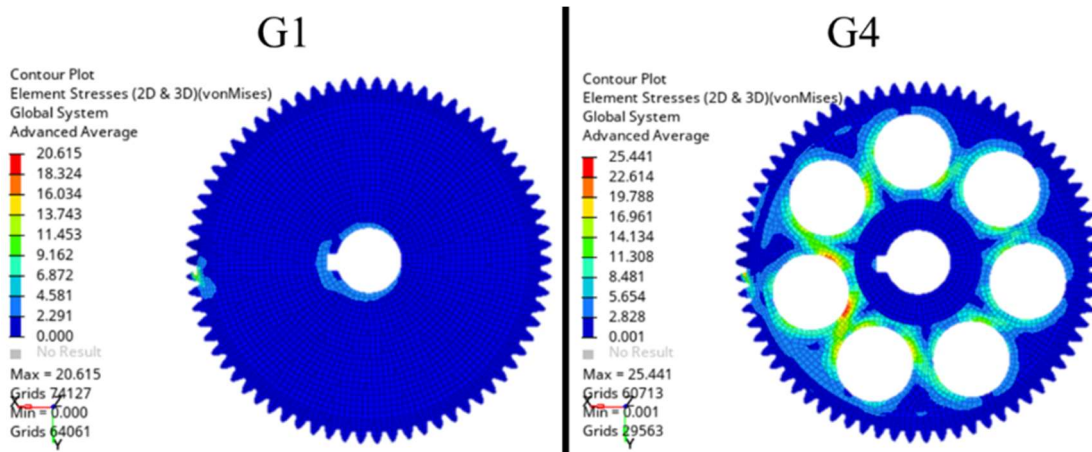


Figure 9: Stress distribution of G1 and G4 models.

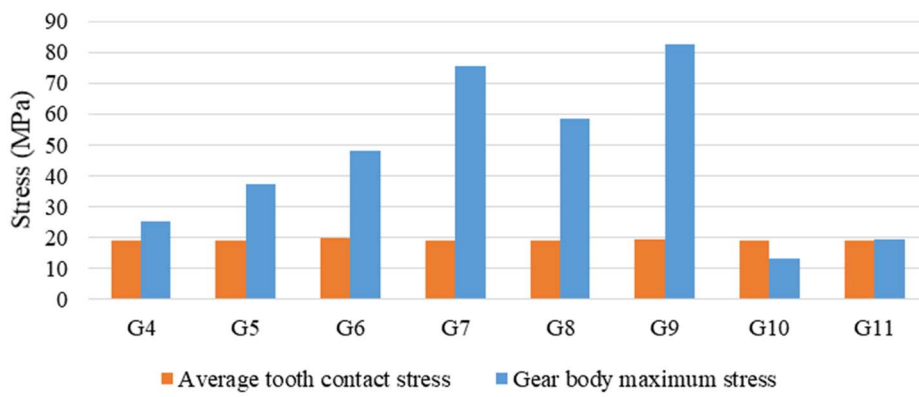


Figure 10: Comparison between the average contact and maximum stress in the gear core.

Comparing the reductions rates of volume, contact, and bending stresses, it was not figured out any clear relation among these three factors, as can be seen in Figure 11.

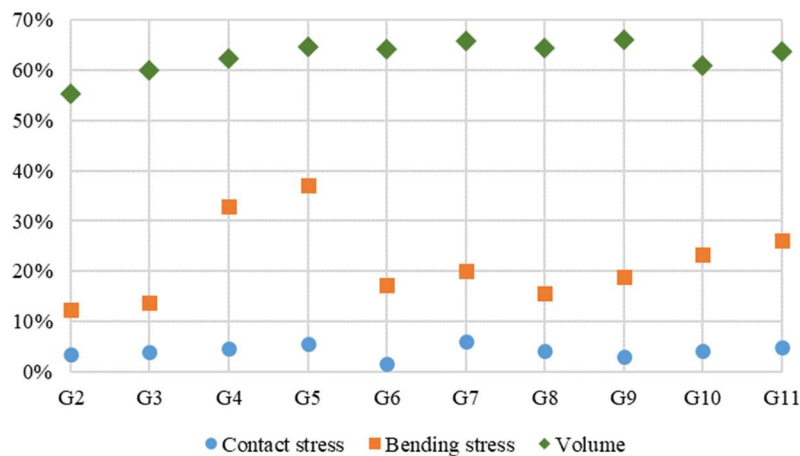


Figure 11: Reduction rates regarding to gear G1.

## 5 Concluding Remarks

This paper evaluated the structural influence of mass relief in spur gears. Altogether, eleven gears geometries were analyzed, being one of them with no reliefs was taken as reference. For each of the geometries, finite element models were built with solid elements and, to emphasize the comparative character, the mesh of the outer rim and the hub of each gear were identical for all geometries.

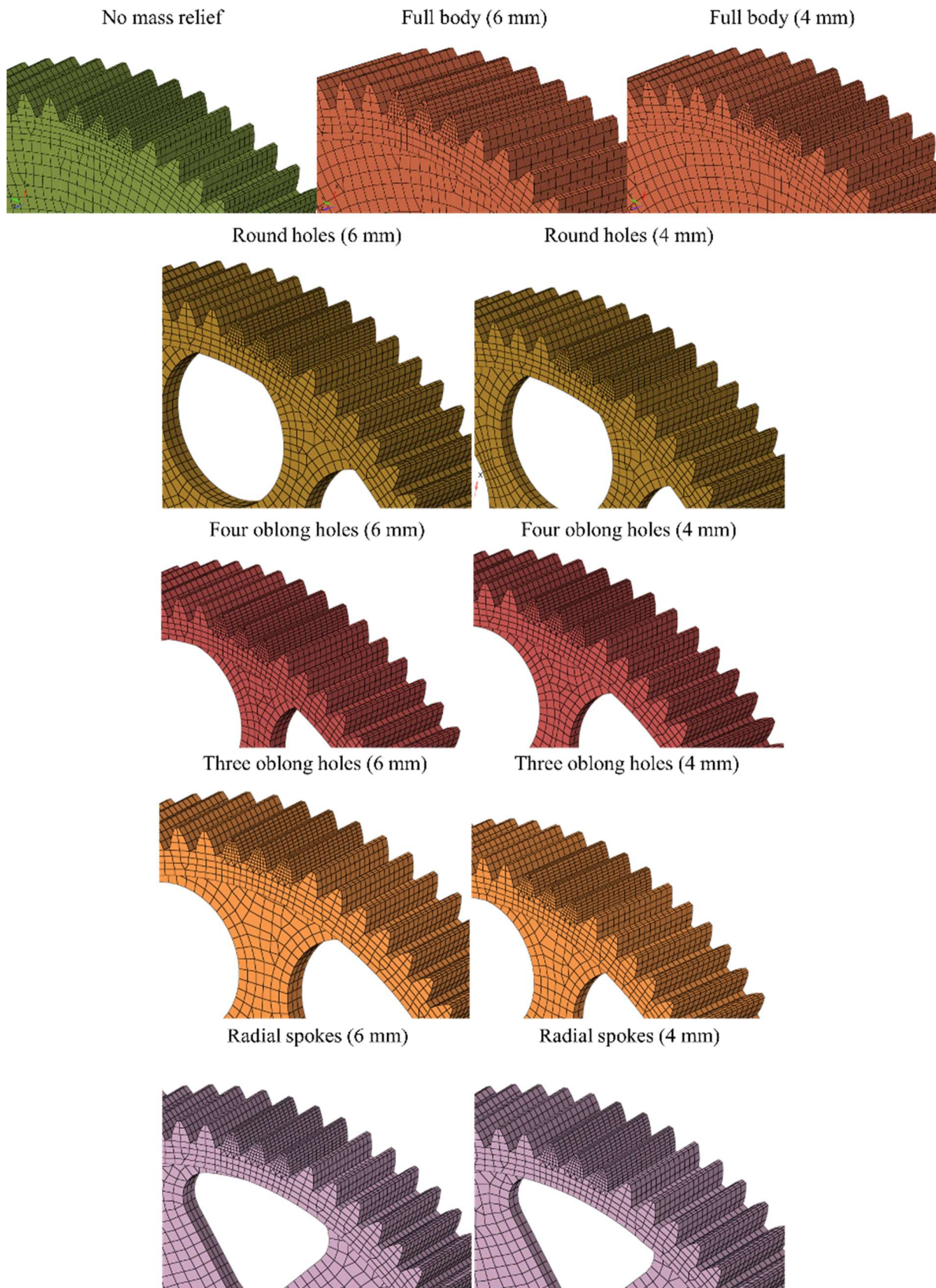
Assessing the stress results, it can be verified that gears with mass reliefs can have benefits not only in material and design savings but also in the design life of the component, since the stresses in the gear teeth decreased in all geometries with mass relief when compared with the reference, with no reliefs.

However, the results obtained also revealed that by decreasing the gear core width, the stresses in this region can increase significantly, which can cause greater losses than the benefits presented before in relation to the gear life, since the gear body would tend to fail before the teeth. Thus, gears shall be designed to reduce the tooth stresses without significantly increasing the stress on the gear core or other regions.

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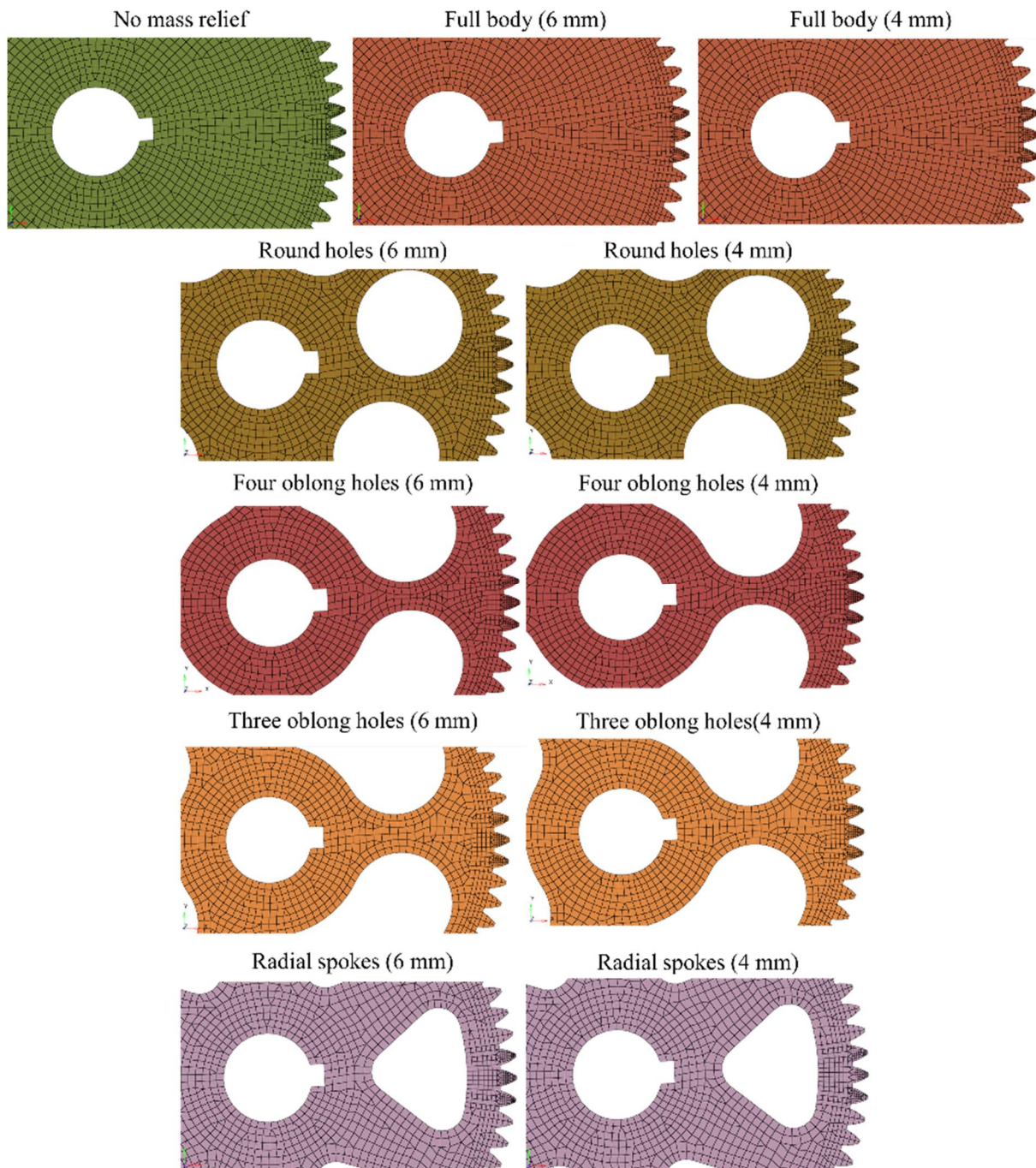
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### Appendix A

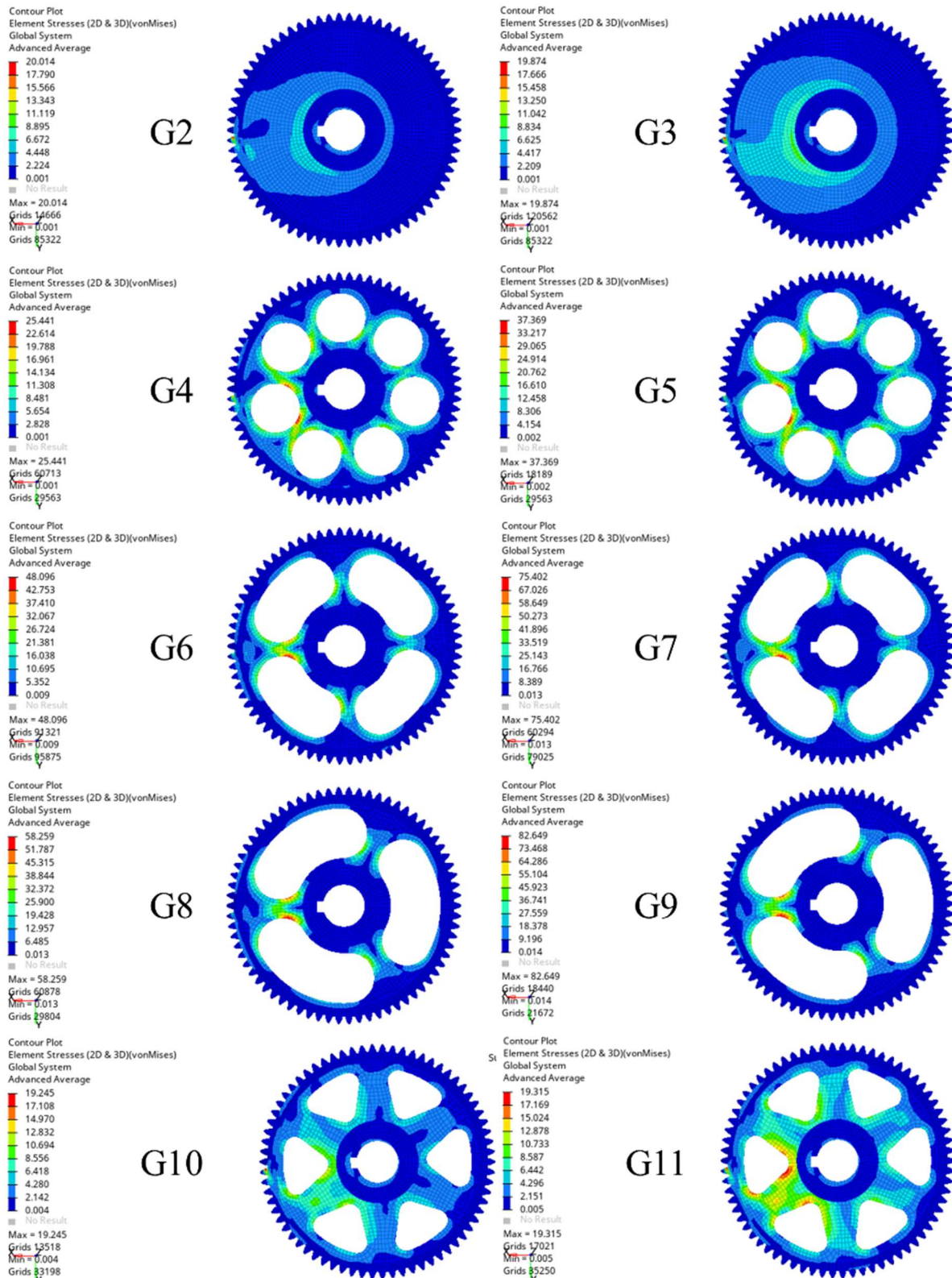




### Appendix B



### Appendix C





### Appendix D

