FAULT DETECTION USING STATE OBSERVERS WITH UNKNOWN INPUT, IDENTIFIED BY ORTHOGONAL FUNCTIONS

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ABSTRACT

In this work a methodology of fault analysis in mechanical systems was developed using Kalman Filter state observes, in which, the input to the observers are identified by Fourier, Legendre and Chebyshev orthogonal functions. The proportional-integral observer is presented to the unknown input identification, this observer can be able to find the unknown inputs of the system and these inputs are used to the fault detection by way Kalman Filter Observer. Here can be seen the methodology of parameters and force identification using only the response of the system thought orthogonal functions. The methodology developed is applied in a composed structure of shake tables from Mechanical Vibrations Laboratory.

WORD-KEY: Filter Observer. Mechanical Systems. Orthogonal Functions.

1. INTRODUCTION

With the increase in production process, are more and more demands of the industries for machines and equipment capable of executing a greater number of functions in less time and in many cases to be capable to act continuously. This leads to the fact of that these systems are submitted the high dynamic forces. Normally these mechanisms are expensive and therefore one of the major concerns of industry is to keep its equipment functioning without necessary breakdowns. With this constant concern, in the last times, we verify much development of new techniques of detection and localization of faults in mechanical systems submitted dynamics loads. In order to guarantee continuo operation of the mechanical systems, they must be supervised and monitored so that the faults are diagnosed and repaired as fast as possible, if not so the disturbance in normal operation can to take the one's a deterioration of the performance of the system or the dangerous situations. Robust observers can reconstruct the states not measured or estimate the motion of the system that can't be measured directly. Thus, faults can to be detected without knowledge of the motion at many points in the system by being able to monitor them through the reconstruction of the states. The existing methodologies using state observers are usually used in control

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problems and detection of possible faults in sensors and in instruments. In this work the state observers are used to faults detection in mechanical systems, using orthogonal functions or Proportional and Integral (PI) observer to estimate the unknown inputs. The Kalman filter observer is used to localization and quantification of the faults. In previous works the identification of the faults using only state observers was possible with the previous knowledge of the inputs [1,2]; in this work the unknown input will be found using the orthogonal functions or PI observers.

Methods of identification of forces or parameters, with the objective of diagnosis of faults in mechanical systems, using orthogonals functions, have been developed since the end of 80's. These methods have used the series of Fourier for the identification of the structural parameters, and developed the inverse methodology for the identification of the forces [3]. Pacheco [4] used some orthogonals functions for parameters identification through of the comparisons between the functions. Pacheco and Steffen [5] published a work where the orthogonals functions were used for identification of parameters in non linear systems. In the work Melo & Morais [6] studied the behavior of the error found in the identification of the parameters varying the number of terms of expansion of the orthogonal functions for some functions [7]. In other work Melo and Morais [1] of different way, had as objective to identify the forces and the parameters of the mechanical systems together, in the described works previous, the identification of the parameters alone was possible with the previous knowledge of the inputs.

It is physical and economically unviable, in some control systems, for transducers to be placed to measure all the variables of state. When analyzing the methodology of state observers, his found that some possess the capacity to reconstruct the inaccessible states, however, the necessary condition for this reconstruction is that the states are observed [8,9]. In the observers described by Luenberger [8] the gain is determined through algorithms of allocation of eigenvalues and eigenvectors of the observer matrix with a certain criterion. A careful analysis must be made so that the speed of estimation, determined for the eigenvalues, is not very great so that sensitivity to the noise in the sensor also is not great. This type of observer corresponds to a deterministic observer. The problem of the noise in the sensor of course leads a stochastic observer who not only handles better the noise in the sensor [10], but also is characterized by having a gain that is optimized as a certain criterion as it will be seen ahead. The optimized observer, or stochastic observer is known as Kalman-Bucy (KF) filter [11]. The filter of Kalman has demonstrated to be useful in many applications [2], however, the interest here is its application with ends to faults detection.

2. ORTHOGONALS FUNCTIONS

A set of real functions $\phi_k(t), k = 1, 2, 3...$ is said to be orthogonal in the interval $[a, b] \in \Re$, if (Equation 1):

$$\int_a^b \varphi_m(t)\varphi_n(t)d(t) = K \qquad \text{Where: } \begin{cases} K = 0 \Rightarrow m \neq n \\ K \neq 0 \Rightarrow m = n \end{cases} \tag{1}$$

So on, the set of functions is said orthonormal the following relation is valid (Equation 2):

$$\int_a^b \phi_m(t)\phi_n(t)dt = \delta_{mn} \tag{2}$$

If δ_{mn} is the Kronecker delta, the set of functions $\phi_k(t)$ is said to be orthonormal and δ_{mn}=0 if $m \neq n$ or δ_{mn}=1 if $m = n$. If a function $\mathrm{f}(t)$ is continuous or partially continuous in the interval $[a, b]$, then $\mathrm{f}(t)$ can be expanded in series of orthonormal functions, as follows (Equation 3):

$$\mathrm{f}(t) = \sum_{n=1}^{\infty} c_n \phi_n(t) \tag{3}$$

Such series, called orthonormal, constitute generalizations of the Fourier series. Admitting that the sum in Equation 3 converges to $\mathrm{f}(t)$, we can multiply both members for $\phi_m(t)$ and integrate them in the interval $[a, b]$, with c_m as the generalized coefficients of Fourier.

The following property, related to the successive integration of the vectorial basis (Equation 4):

$$\int_0^t \underbrace{\ldots}_{n\,\text{times}} \int_0^t \{\phi(\tau)\}(d\tau) \cong [P]^n \{\phi(t)\} \tag{4}$$

Where [*P*] $\in \Re^{r,r}$ is a square matrix with constant elements, called operational matrix [3], and $\{\phi_m(t)\} = \{\phi_0(t)\ \phi_1(t)\ ...\ \phi_r(t)\}^T$ is the vectorial basis of the orthonormal series. In fact, if a complete vectorial base is regarded, or on other world, if the series are not truncated, the relation obtained in Equation 4 is an equality. However, in practice, it becomes not suitable, due to the high order of the matrix [*P*] obtained. A following sections, the vectorial basis and

the operational matrix related to each type of orthogonal function considered in this paper are briefly reviewed [4].

Quadro 1: Fourier series

Vectorial basis in the interval $[0,T]$	Operational matrix of integration
$\{\varphi(t)\}=\{\varphi_0(t) \quad \varphi_1(t) \quad ...\varphi_s(t) \quad \varphi_1^*(t) \quad ...\varphi_s^*(t)\}^T$ $\varphi_n(t)=\cos\frac{2n\pi t}{T} \quad , n=0,1,2,...,s$ $\varphi_n^*(t)=\sin\frac{2n\pi t}{T} \quad , n=1,2,...,s$ $r=2s+1$	$[P]=\begin{bmatrix} \frac{T}{2} & \{0\}_{1xs} & -\frac{T}{\pi}\{\tilde{e}\}_s^T \\ \{0\}_{sx1} & [0]_{sxs} & \frac{T}{2\pi}[\tilde{I}]_{sxs} \\ \frac{T}{2\pi}\{\tilde{e}\}_s & -\frac{T}{2\pi}[\tilde{I}]_{sxs} & [0]_{sxs} \end{bmatrix}_{rxr}$ $\{\tilde{e}\}=[1 \quad 1/2 \quad 1/3 \quad \dots \quad 1/s]$ $[\tilde{I}]_{sxs}=diag\{1 \quad 1/2 \quad 1/3 \quad \dots \quad 1/s\}$

T = Period of sampling and s = number of terms of Fourier in sines and cosines

Quadro 2: Legendre polynomials

Recursive formula on the interval $t\in[0,\mathrm{t_f}]$	Operational matrix of integration
$(n+1)p_{n+1}(t)=(2n+1)\left(\frac{2t}{t_f}-1\right)p_n(t)-np_{n-1}(t)$ $,\mathrm{n}=1,2,3,\dots,r-1$ $p_0(t)=1$ $p_1(t)=2t/t_f-1$	$[P]=\frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ -\frac{1}{3} & 0 & \frac{1}{3} & 0 & \cdots & 0 \\ 0 & -\frac{1}{5} & 0 & \frac{1}{5} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{-1}{2r-3} & 0 & \frac{1}{2r-3} \\ 0 & 0 & \cdots & 0 & \frac{-1}{2r-1} & 0 \end{bmatrix}$

r = number of terms truncated

Quadro 3: Chebyshev polynomials

Recursive formula on the interval $t\in[0,\mathrm{t_f}]$	Operational matrix of integration
$T_{i+1}(t)=2\left(\frac{2t}{t_f}-1\right)T_i(t)-T_{i-1}(t)$ $i=1,2,\dots,r-1$ $T_0(t)=1$ $T_1(t)=\frac{2t}{t_f}-1$	$[P]=\frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ -1/4 & 0 & 1/4 & 0 & \dots & 0 & 0 & 0 \\ -1/3 & -1/2 & 0 & 1/6 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{(-1)^{r-1}}{(r-1)(r-3)} & 0 & 0 & 0 & \dots & \frac{-1}{2(r-3)} & 0 & \frac{1}{2(r-1)} \\ \frac{(-1)^r}{r(r-2)} & 0 & 0 & 0 & \dots & 0 & \frac{-1}{2(r-2)} & 0 \end{bmatrix}$

r = number of terms truncated

2.1. Identification of mechanical systems through orthogonal functions

The proposed identification method can exploit either on the free or forced time domain responses, as functions of displacements, velocities or accelerations. Once the formulations for these three kinds of responses are quite similar [6]. Only the formulation for forced systems in terms of displacements, will be presented.

The development of the method starts on the equation of motion of a forced mechanical system of *N* degrees of freedom (Equation 5):

$$[M]\left\{\ddot{x}(t)\right\}+[C]\left\{\dot{x}(t)\right\}+[K]\{x(t)\}=\{f(\mathrm{t})\} \tag{5}$$

Where [M], [C] and [K] are the inertia, damping and stiffness N-order matrices respectively; $\{x(t)\}$ is the vector of displacement time responses and $\{f(t)\}$ is the vector of exciting forces.

Integrating Equation 5 twice in the interval [0,t], it can obtain the Equation 6:

$$\begin{aligned}&[M]\left(\{x(t)\}-\{x(0)\}-\{\dot{x}(0)\}t\right)+[C]\left(\int_0^t\{x(\tau)\}d\tau-\{x(0)\}t\right)+[K]\int_0^t\int_0^t\{x(\tau)\}d\tau^2=\\&=\int_0^t\int_0^t\{f(\tau)\}d\tau^2\end{aligned} \tag{6}$$

The signals $\{x(t)\}$ and $\{f(t)\}$ can be expanded in the truncated series of *r* orthogonal functions as follows (Equation 7):

$$\{x(t)\}=[X]\{\phi(t)\} \qquad \text{and } \{f(t)\}=[F]\{\phi(t)\} \tag{7}$$

where: $[X]\in\Re^{N,r}$ is the matrix of the coefficients of expansion $\{x(t)\}$

$[F]\in\Re^{N,r}$ Is the matrix of the coefficients of expansion $\{f(t)\}$

Substituting Equation 7 in Equation 6 and applying the integral property given by Equation 4, the following system of algebraic equations can be obtained the Equation 8 [1].

$$\left[\begin{array}{cccc}[\mathrm{M}] & -[\mathrm{M}]\{x(0)\} & \left\{-[\mathrm{M}]\{\dot{x}(0)\}-[\mathrm{C}]\{x(0)\}\right\} & [\mathrm{C}] \quad [\mathrm{K}]\end{array}\right]\left[\begin{array}{c}[X] \\ \{e\}^T \\ \{e\}^T[P] \\ [X][P] \\ [X][P]^2\end{array}\right]=[F][P]^2 \tag{8}$$

Change the Equation 8, it can be written according to Equation 9:

$$\left[\begin{array}{ccc}[F] & [M]\{x(0)\} & [M]\left\{\dot{x}(0)\right\}+[C]\{x(0)\}\end{array}\right]\left[\begin{array}{c}[P] \\ \{e\}^T \\ \{e\}^T[P]\end{array}\right]=[M][X]+[C][X][P]+[K][X][P]^2 \tag{9}$$

And so, the Equation 8 and the Equation 9 can be represented as (Equation 10):

$$[H][J]=[E] \tag{10}$$

Identifying H to the Equation 8 we can to determine the structural parameters of the system. And doing the same to the Equation 9 we can determine the system inputs.

3. GENERAL STRUCTURE OF THE STATE OBSERVER: KALMAN FILTER

Considering a linear system, invariant and observable in the time (Equation 11):

$$\bar{S}: \begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) + L\,\xi(t) \\ y(t) = C\,x(t) + \eta(t) \end{cases} \tag{11}$$

where: *x(t)* is the state vector *n x 1*, *u(t)* is the input vector *p x 1*, *y(t)* is the output vector *k x 1*, *A* is the matrix of system *n x n* (dynamic matrix), *B* is the matrix of distribution *n x p* (matrix of inputs), *C* is the matrix of measures *k x n*, being *n* the order of the system, *p* the number of inputs u(t), and *k* the number of outputs *y(t)*. The vector ξ is called noise of excitement in the state and represents a disturbance in the system and the vector η is called noise in the sensor [12]. Due to stochastic nature of the vectors ξ and η, in the Kalman Filter, they have certain statistical properties, corresponding to the white Gaussian noise, stationary (invariant in the time) and not correlated between itself.

Now we can to define the matrixes $\Xi \text{ and } \Theta$, they are called of intensity of the noise $\xi \text{ and } \eta$, respectively, and matrices symmetrical and positive are defined (Equation 12):

$$\Xi = \Xi^T \geq 0,\ \Theta = \Theta^T > 0 \tag{12}$$

Given the assumptions mentioned above, the problem of optimum estimate of the state vector x in presence of white noises (as vectors of state as the measured variable) can be formulated to find optimum value (filter of Kalman) that it generates an estimate $\bar{x}$ for the real state vector x, so that minimizes the covariance of the error estimation $e(t) = \bar{x}(t) - x(t)$ (Equation 13):

$$\Im_{KF} = \mathrm{E}\left[e(t)e^T(t)\right] \tag{13}$$

In according to restriction Equation 11, Kalman and Bucy had proved that the best structure for the Kalman filter (among all the possible structures, linear and nonlinear) when the dynamics of the system is linear and the noises are white and Gaussians is the following one (Equation 14):

$$\bar{S}_{KF} : \begin{cases} \dot{\bar{x}}(t) = A\ \bar{x}(t) + B\ u(t) + \mathrm{K}_{KF}\left(y(t) - C\ \bar{x}(t)\right) \end{cases} \tag{14}$$

In which K_{KF} is the matrix of the state observer, $\{\bar{x}(t)\}$ is the state vector of the observer.

3.1. Filter Algebraic Riccati Equation (Fare)

The solution of the optimization problem can be found in literature. Since in the present work the main interest is the application of the control methodologies, we present here without test the solution for this problem. The optimum gain K_{KF} for the Kalman filter is given by the following relation (Equation 15):

$$\mathrm{K}_{KF} = S_{KF} C^T \Theta^{-1} \tag{15}$$

In which S_{KF} is defined like a symmetrical and positive matrix satisfying the Riccati equation for the Kalman filter (FARE) (Equation 16):

$$S_{KF}A^T + A\ S_{KF} + L\ \Xi\ L^T - S_{KF}C^T\Theta^{-1}C\ S_{KF} = 0 \quad (16)$$

4. METHOD OF THE STATE OBSERVERS WITH UNKNOWN INPUTS

The state observers where all the inputs of the system must be known and available have great utility in the case where only one input to the control system. In the cases where the system is submitted the unknown inputs or disturbance which cannot be measured or the measurement is very difficult or simply impossible, the performance of observer can very be poor. In this work was developed a methodology of diagnose of faults using observers of state in which its input is considered unknown or partially unknown in which the Proportional and Integral observer is used to estimate the unknown inputs, in which, the gain of this observer is determined for the gain given by the Kalman Filter. After the identification of the unknown inputs these are used for the detection of possible faults that are occurring in the systems. For this, the Kalman Filter was used to generate unknown states.

A very convenient representation for systems with these characteristics is as indicated for the following equation (Equation 17):

$$S:\begin{cases}\dot{x}(t) = A\,x(t) + B\,u(t) + B_d\ v_d(t)\\ y(t) = C\,x(t)\end{cases} \quad (17)$$

In which:

$x(t)$ is a state vector *n x 1*, $u(t)$ is a input vector *r x 1*, $y(t)$ is a output vector *m x 1*, $v_d(t)$ is a vector of disturbance or unknown input *p x 1*, A is a matrix of system *n x n* (dynamic matrix), B is a matrix of distribution *n x r* (matrix of input), C is the matrix of measures *m x n* and B_d is the matrix distribution of disturbance *p x n*, being *n* the order of the system, *r* the number of inputs *u(t)*, *m* the number of outputs *y(t)* and *p* the number of disturbance $v_d(t)$.

The estimate problem to the state of a linear and invariant system in the time with known and unknown inputs has been subjects of researching in the last decades and with considerable importance because in real system, there are many situations where the disturbance are present or some inputs are inaccessible, then a conventional observer which all the inputs are known can not be used. This way, an observer capable of estimate the state for linear system with partially unknown inputs, not sensible to disturbance, can be of great utility.

The idea is projecting an observer who is capable of estimate the disturbance v_d. The FIGURE 1 suggests the function of this observer.

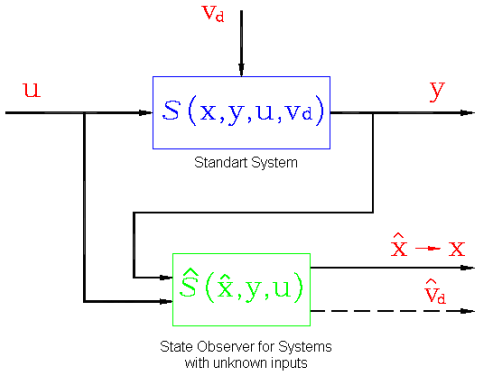


Figure 1: Observer with unknown inputs

4.1. Modeling of the Observer with Unknown Inputs

In according this approach, we verify that the dynamics of the disturbance vector satisfies the differential equations following (Equations 18 and19)):

$$v_d(t) = c_d w(t) \tag{18}$$

$$\dot{w}(t) = A_d w(t) \tag{19}$$

Which *w* represents the disturbance state contained in the matrix A_d and the matrix C_d indicates like the disturbance is dependent of this state. The choice of these matrices depends on the kind of the disturbance. Thus, for example, in the case where the disturbance v_d *is* constant, a convenient choice for this is that matrix $A_d = 0$ and $C_d = I$ (I is a identity matrix). Arranging the Equation 17 with Eqtions 18 and 19 we get a increased model of state (Equation 20):

$$S_a : \begin{cases} \dot{x}_a(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} A & B_d C_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v_d(t) \end{bmatrix} \end{cases} \tag{20}$$

It is verified in the equation above that *w* is not controllable through of *u*. But, in general, it is observable [11] and with this, is possible to project an observer for this system that

estimate as variable *x* as *w*. Thus, an observer of full order for this new system will be (Equation 21):

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{w}}(t) \end{bmatrix} = \begin{bmatrix} A & B_d C_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} (y(t) - C\,\hat{x}(t)) \tag{21}$$

In which the matrix $K = \begin{bmatrix} K_1^T & K_2^T \end{bmatrix}^T$ to guarantees stability of the observer. In the same form, we have (Equation 22):

$$\hat{S}_{ed/ds} : \begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + B_d C_d \hat{w}(t) + K_1 (y(t) - C\hat{x}(t)) \\ \dot{\hat{w}}(t) = A_d \hat{w}(t) + K_2 (y(t) - C\hat{x}(t)) \end{cases} \tag{22}$$

4.2. Proportional-Integral Observer

When the spectrum of disturbance does not contain high frequencies, the observer of the section (2) can be used considering $A_d = 0$ and $C_d = I$ getting a simplification in the model. In this case the corresponding part to the estimation the disturbance vector becomes in a bank of integrators and the corresponding part to the estimation the state vector becomes in proportional and integral to the residual: $y(t) - C\hat{x}(t)$. This observer is called proportional-integral or PI and has superior properties comparing with the proportional observer of full order. The observer proportional-integral is capable estimate any disturbance (constant, linear and nonlinear) but it has to be slower than the constant of time of integral action and the number of measurements can't be minor that the number of disturbance. Increasing the integral gain is possible to reject the faster disturbance, however, this has a negative effect decreasing the stability of the observer. Using the Equation 22, we have for the case of proportional-integral observer (Equations 23 and 24):

$$\hat{S}_{pi} : \begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + B_d v_d(t) + K_P (y(t) - C\hat{x}(t)) \\ \dot{\hat{v}}_d(t) = K_I (y(t) - C\hat{x}(t)) \end{cases} \tag{23}$$

Or equivalent: $$\hat{S}_{pi} : \{\dot{\hat{x}}_a(t) = A_a \hat{x}_a(t) + B_a u(t) + K_a (y(t) - C_a \hat{x}_a(t)) \tag{24}$$

In which: $\hat{x}_a = \begin{bmatrix} \hat{x} \\ v_d \end{bmatrix}$, $A_a = \begin{bmatrix} A & B_d \\ 0 & 0 \end{bmatrix}$, $B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $C_a = \begin{bmatrix} C & 0 \end{bmatrix}$, $K_a = \begin{bmatrix} K_P \\ K_I \end{bmatrix}$

The necessary and enough condition for the existence of the observer is that the pair (A_a, C_a) has been, in the least, observable, thus it is possible to place the eigenvalues of the following matrix of the complex plan:

$$\hat{A}_a = A_a - K_a C_a = \begin{bmatrix} A - K_P C & B_d \\ -K_I C & 0 \end{bmatrix} \tag{25}$$

In this work the gain of observer PI is determined by the gain gotten for Kalman Filter presented in section (3.1).

4.3. Example

Following is presented an example of determination of an unknown input in a robotic arm as shown in FIGURE 2.

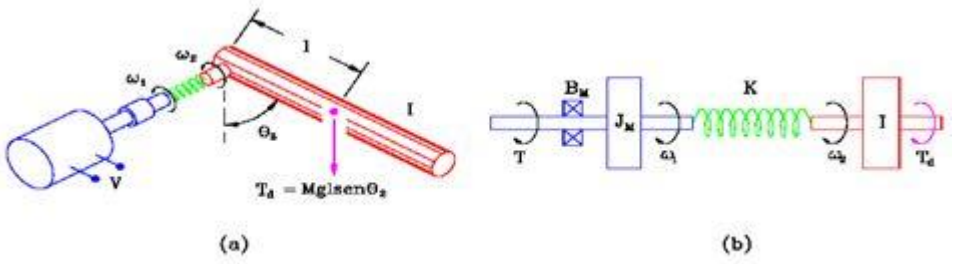


Figure 2: flexible arm of a robot with unknown input (disturbance) from the weight.

One mathematical model can be represented by the state Equation 17 in which the matrices are given by:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K/J_M & K/J_M & -B_M/J_M & 0 \\ K/I & -K/I & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ K_e/J_M \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\text{and } x = \begin{bmatrix} \theta_1 & \theta_2 & \omega_1 & \omega_2 \end{bmatrix}^T$$

In which:

$\theta_1(t)$: Angular displacement of the robot arm ($\theta_1(0)$=15°)

$\theta_2(t)$: Angular displacement in output of the reduction box ($\theta_2(0)$=15°).

$\omega_1(t)$: Angular speed of the robot arm ($\omega_1(0)$=0).

$\omega_2(t)$: Speed in the output of the box of reduction ($\omega_2(0)$=0).

V: Voltage of armor of motor DC (= square shaped Wave of 5 V and 3 *rad/s)*.

I: Inertia of the arm robot (= 0.4Kg *m2).*

K: Torsional stiffness of the spring (= 1 *N m/rad)*.

J_M*:* Inertia equivalent of the motor including reduction box (= 0,0424 *kg m2*)

B_M*:* Viscous friction in the motor (= 0,0138 *N m s/rad).*

K_e: Momentum Gain for the motor (= 0,0403 *N m/V*).

To simulate the system was used the Runge Kutta method, in which it is considered as output unknown a nonlinear force from the weight of the arm and equal the T_d = M g l sin (θ_2) with M = 1 *Kg,* g = 9,8 *Kg ms* $^{-2}$ and l = 0,3 *m*. The variable of state estimated by observer PI does not consider the force nonlinear and the PI observer can estimate this force (disturbance). For PI observer, the nonlinear force is considered as being an interferential input to the system. In the FIGURE 3 it is presented the real input and the estimate for the observer.

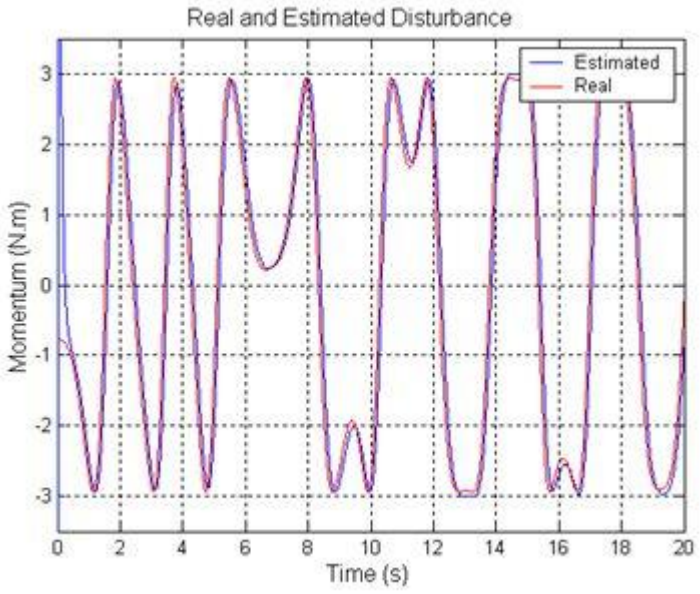


Figure 3: Unknown input estimated through PI observer

4.4. Project of State Observers

The project of a system is presented in the FIGURE 4 functioning with state observers, where if verifies the known excitement force *u(t)*, unknown inputs v_d *(t),* the measured outputs *y(t)*, the observers PI used to identify the unknown input, the global and robust observers to the parameters subject to faults s1…, sn and a unit of logical decision. The observer of global state is responsible for the detection of the fault, the robust state observer is responsible for the location of the same one. The global observer is a copy of the original

system, and analyzes all the system detecting possible faults. The robust state observer can detect the fault if this occurs in the parameter for which it was projected. We have to project a bank of robust observer, each one in relation to a parameter to be monitored, to become possible a good location of fault.

When the system is functioning adequately, without indications of faults, the observer of global state answers equal the real system. When one component of the system in question starts to fail, the state observer feels the influence of this process quickly. The objective is using this effect of the state observer to locate and to quantify the fault in the mechanical system. The global and robust observers are modeled, in this work, using the methodology of the Kalman Filter because with this the noise in the system is better worked. They are put in a bank of observers and the RMS values of the differences between the real signs in displacement (measured) and the generated for the observers are analyzed in a unit of logical decision that it analyzes the trend of the progression of the fault and sets in motion, when will be necessary, an alarm system. The alarm system can be initialized when we have a parameter variation. This is on line process and the model of PI observer must be changed during all the process where the fault is occurring, with possibility to identify the disturbance with good accuracy.

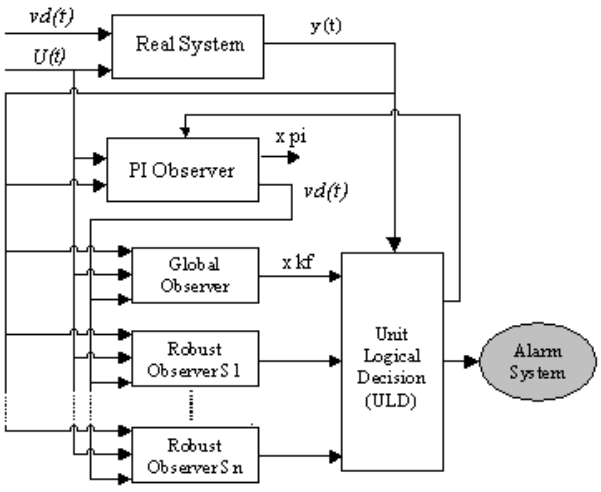


Figure 4: System of Robust Observation.

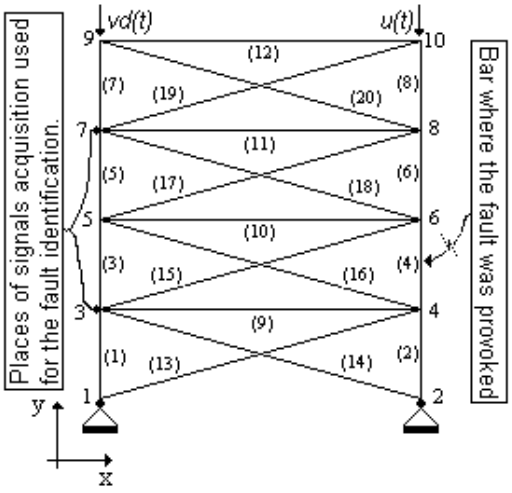


Figure 5- Truss Structure with 20 bars

5. SIMULATION AND RESULTS FOR A TRUSS STRUCTURE WITH 20 BARS

To validate the methodology of identification and location of faults applied the mechanical systems using the state observer, filter of Kalman, with forces unknown identified through PI observers, was simulated a truss structure with 20 bars as shown in the FIGURE 5. For this, we used the finite elements method to be able to simulate the structure, where each bar

represents a composite element for two joins and each join has two degrees of freedom (dof) being displacement in x and y. Considering that the structure has restrictions in the joins 1 and 2, we have a system with 16 dof, as shown in the FIGURE 5, the system was excited in the join 9 and 10 in the direction of y with harmonic forces of 300 N and 500 N and frequencies of 250 rad/s and 3700 rad/s, in that order. The force applied in join 9 is considered unknown and will be determined by observer PI, as be seen in the FIGURE 7.

All the elements that compose the truss are isoperimetric with the following properties: ρ = 7850 kg/m3, E = 200GPa, height = 2.0cm, width = 3.0cm. All the bars in the x direction have 2.0m and in the y direction have length equal 0.5m. We considered during the simulation a low proportional damping the matrix of mass and the stiffness of the system given for: *C=1.0e-10*K+1.0e-04*M*. The output of this system was gotten through the method of Runge-Kutta of fourth order with 4096 points in the interval of 1.0 s, of this form, was used only the output of the displacement in the direction x in joins 3 and 7, with this we initiated the identification and location process of the fault which the structure was submitted simulating to fault we used a reduction in the area of bar four of 30%. To validate the robustness of the Kalman Filter for presence of noises in the signs, it was added, to the input, a white noise with energy equal 5% of the value of the energy of the input sign *u(t)*.

The bank of robust observers is generated for the parameters subjected to the faults with percentile variation of 10% in the area of each bar. Was considered, in this work, that all the bars of the system are susceptive the occurrence of a possible fault. In the FIGURE 6 the is presented the inverse values of differences RMS found between the “measured” sign in the structure and the signs generated for the global observers (0% of fault) and for the robust observers, reducing in 10% the value of each subject parameter to fault. In FIGURE 6 could be located and be quantified the fault provoked in bar 4 with 30% of reduction in this parameter.

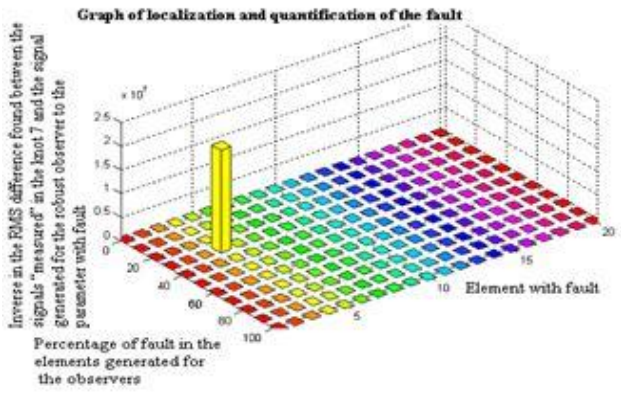


Figure 6: Bank of robust observers generated

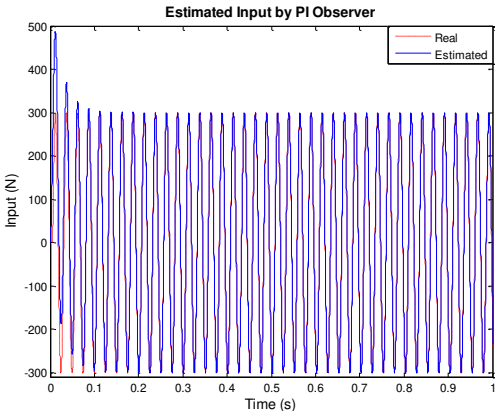


Figure 7: Estimated input by PI observer

6. EXPERIMENTAL RESULTS

A dynamic system of shake tables constituted of metallic stainless steel blades constructed and these blades represent the stiffness to the system; plates of aluminum for construction of the tables and rubber to simulate viscous damping. The rubbers are fixed between the blades, as if it can observe in the FIGURE 8, the structure was modeled like a system of three degrees of freedom with discrete parameters. The structural parameters had been determined using techniques of experimental modal analysis, for this, the parameters of mass, damping and stiffness had been determined for each table separately. In the TABLE 1 the results are presented. Being unattached the three tables, it was gotten excited inferior table with a harmonic force and the acquisition of the signals during 1,0 *s* and with 2048 points in this interval, for this, using DASYLAB software with four channel for signals acquisition, where the three first channel had been used for acquisition of the signals of displacement and the last channel for determination of the excitement force. The signal used was of displacement, therefore the answers measured for the accelerometers had been integrated two times, using, for this, the Conditioner/Amplifier of signals *Nexus of the BRUELL* that have this function.

Table 1: Space parameters identified for the structure from analyze modal experimental classic

Table	Inferior	Intermediary	Superior
M (kg)	6.644	4.619	1.889
K (kN/m)	275.367	114.489	104.993
C (N s/m)	100.042	36.360	29.660

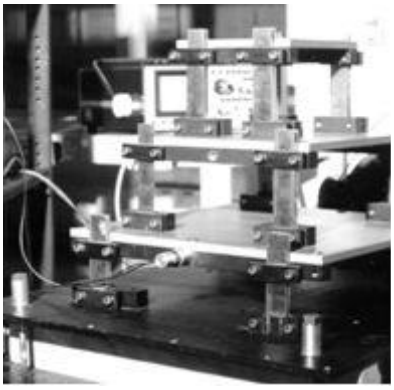


Figure 8: Test Rig-Vibratory System

The fact to measure the excitement force of the system had the objective of comparison and verification of the method efficacy. In the FIGURE 9 can be verified the inputs estimated through the orthogonal functions by way the Fourier, Legendre and Chebyshev methods, as can be seen in the Equation 9 during the identification of the inputs had been used 100 terms of expansion as seen in the work [6].

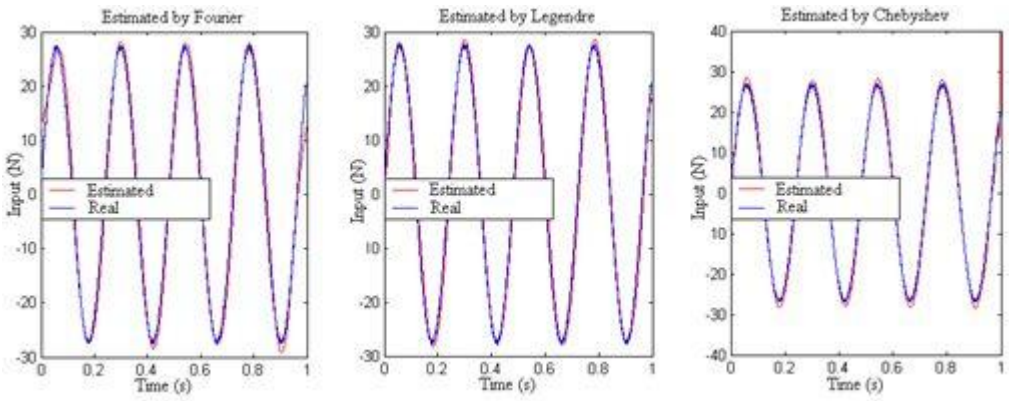


Figure 9: Input identified through Fourier, Legendre and Chebyshev methods, respectively.

For the fault detection was used only the output of the displacement measured in the inferior table, with this, we initiated the identification and location process of the fault a plate of the superior table was removed and was verified a reduction of 8,9% in the stiffness of this parameter. The bank of robust observers is generated for the parameters subjected to the faults with percentile variation of 1% in the stiffness of the tables. In the FIGURE 10 are presented the inverse values of differences RMS found between the measured sign in the structure and the signs generated for the global observers (0% of fault) and for the robust

observers, reducing in 1% the value of each subject parameter to fault. In FIGURE 10 could be located and quantified the fault provoked in the superior table in the region of 9% of reduction in the stiffness by way the inputs identified through Fourier, Legendre and Chebyshev methods, in that order.

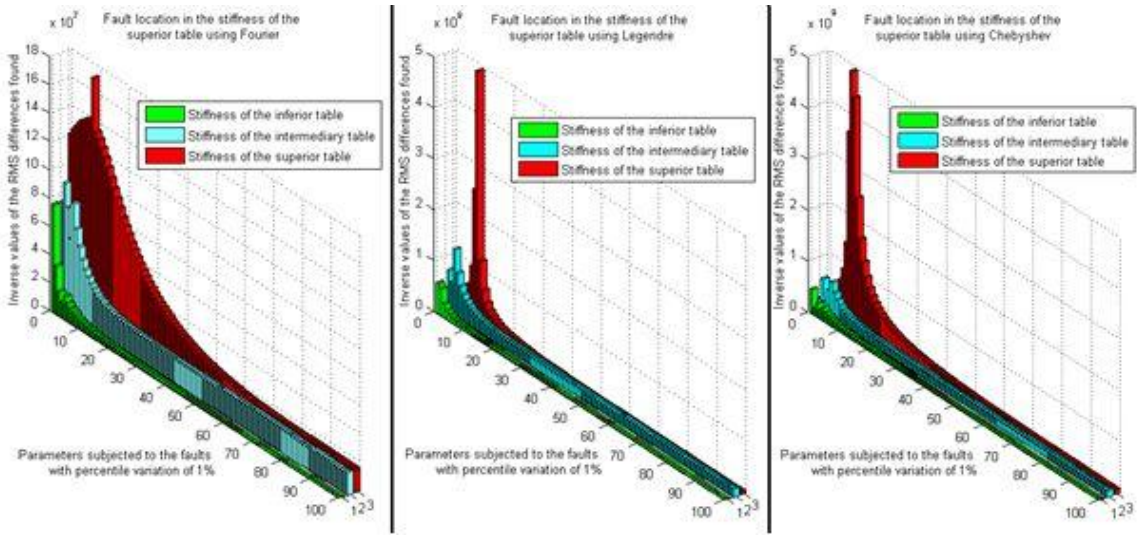


Figure 10: Fault detection and location through Fourier, Legendre and Chebyshev methods.

The good result gotten during the force and parameters identifications is, in large part, due to high simplicity and linearity of the analyzed system.

7. CONCLUSIONS

In this work was developed a methodology of diagnose of faults using state observers. It was used the Kalman Filter to the construction of bank of observers, and in this case the observer needs that all the inputs are known or with white noise because it is the only kind of interference that we can used to project the Kalman Filter. Here the inputs are identified by the way orthogonal functions or Proportional and Integral observer. With this gain the states identified by PI observer reject the inputs unknown. It was presented a robotic arm, in which, it was possible to identify the external force due the mass of the arm using PI observer. A diagnose of fault was carried using a truss structure of 20 bars in which were considered two inputs, being that one was unknown and identified by PI observer. The experimental validation of the methodology was carried through from a simple system of three degrees of freedom, with force estimated using orthogonal functions. The fault was well identified for all

functions used. When we consider the time computational necessary for the assembly of the bank of robust observers to the parameters subject to faults, this time is relatively high, but in the practical the bank of state observers is assembled only one time, of this form during the acquisition of signs on-line in a structure is not necessary to assemble the bank of observers given who this must be assembled previously. With this, we can conclude this method can be used during processes of detection of faults on-line.

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